EFFECT OF THE WIND AND RAIN ON THE REAERATION COEFFICIENT IN LAKES

by

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RESUMEN

En este estudio se demuestra que el coeficiente de reaeración superficial en un lago o laguna depende de la velocidad del viento, y de la tasa y potencia de la lluvia.

También se establece que la adición directa de oxígeno, a partir del oxígeno saturado de las gotas de lluvia, puede ser un factor importante en la reaeración.

RESUMEN

1. SUMMARY OF THE REPORT

1.1 Summary of results

This report presents the results of a study carried out to determine the effects of wind action and rainfall on the surface reaeration coefficient in lakes and lagoons. The main results of the study are listed below.

1. The oxygen transfer coefficient due to wind action, K_o, can be determined from the following equation

$$K_o = 10^{-6} \left[8.43 \sqrt{U} - 3.67\, U + 0.43\, U^2\right] \qquad (1.1.1)$$

where U is the velocity of the wind. The units of K_o and U are m/s. The surface reaeration coefficient due to wind action, $K_{o,2}$, is related to the oxygen transfer coefficient, K_o, by the following equation

$$K_o = K_{o,2}\, H \qquad (1.1.2)$$

where H is the depth, in m.

2. The velocity, V, of a spherical particle falling through a fluid is given by the equation

$$V = \sqrt{\frac{4}{3} \frac{g(\rho' - \rho) D}{\rho C_d}} \qquad (1.1.3)$$

in which D is the diameter of the particle, g is the gravitational acceleration, ρ' and ρ

are the densities of the particle and the fluid, respectively, and C_d is a drag coefficient. The magnitude of C_d depends on the Reynolds number, $Re = \rho\ V\ D/\mu$, where μ is the viscosity of the fluid. For values of Re less than about 1.0, eq. *1.1.3* reduces to

$$V = \frac{g\,(\rho' - \rho)\,D^2}{18\,\mu} \tag{1.1.4}$$

This result is generally identified as Stokes law.

3. The velocity, V (cm/s), of a water drop of diameter, D (mm), falling in air can be computed from the following equation

$$V = 5.49\,D^3 - 88.80\,D^2 + 491.84\,D - 16.60 \tag{1.1.5}$$

for the range of diameters from 0.1 to 5.8 mm. Because of the change of the density of air, rain drops have larger fall velocities at high elevations than at low elevations. At sea-level, the maximum values of the diameter and velocity of a rain drop are approximately 6.0 mm and 920 cm/s, respectively. Larger values are prevented because of an instability which causes the drop to fragment into smaller droplets.

4. The number of drops per unit volume with diameters in a drop diameter interval, ΔD, can be described by an exponential-type equation

$$N = N_o\ exp\ (-\ \lambda\ D) \tag{1.1.6}$$

where

$$\lambda = 41.0/r^{0.21} \tag{1.1.7}$$

in which r is the rainfall rate in mm/h. These results were obtained from experiments conducted by other investigators.

5. The number of drops crossing a unit horizontal area per unit time is

$$n(drops/s\text{-}m^2) = N\ V \tag{1.1.8}$$

The volumetric concentration of drops is

$$C\,(cm^3/m^3) = N\ v_o \tag{1.1.9}$$

where v_o is the volume of a drop of diameter, D. The rainfall rate corresponding to a drop diameter interval is

$$r'\ (cm/s) = C\ V = n\ v_o \tag{1.1.10}$$

The kinetic energy of a single rain drop is

$$E\,(\mu J) = \frac{10^{-1}}{2}\ \rho' v_o\ V^2 \tag{1.1.11}$$

The energy flux or power of the drops in a drop diameter interval is

$$P(\mu J/s\text{-}cm^2) = En = \frac{10^{-1}}{2}\ \rho' n\, v_o V^2 = \frac{10^{-1}}{2}\ \rho' r'\ V^2 \tag{1.1.12}$$

The total values of the above-indicated rainfall parameters are described by equations of the form

$$N_T,\ n_T,\ C_T,\ P_T = a_i\ r^{m_i} \tag{1.1.13}$$

Values of a_i and m_i corresponding to sea-level conditions are

Parameter	a_i	m_i
N_T, drops/m^3	1830	0.22
n_T, drops/s-m^2	1950	0.40
C_T, cm^3/m^3	0.083	0.85
P_T, μJ/s-cm^2	0.239	1.26

The rainfall rate, r, is expressed in mm/h. The effect of elevation on the power of a rainfall is given by the following equation

$$P = (0.239 + 0.103 \times 10^{-4}\ z)\ r^{1.26} \tag{1.1.14}$$

in which z is the elevation, in m, above sea-level.

6. The oxygen transfer coefficient due to rainfall, K_r, can be determined from the following equation

$$K_r = b_1\ P \tag{1.1.15}$$

where $b_1 = 2.83\ cm^2/N$; the units of K_r are cm/s.

7. The total oxygen transfer coefficient, K (cm/s), due to the combined effects of wind action and rainfall can be computed from the expression

$$K = (K_o - (K_o\ K_r/K^*) + K_r) \tag{1.1.16}$$

in which $K^* = 0.0246$ cm/s. This equation can be written in the following dimensionless form

$$\left(1 - \frac{K}{K^*}\right) = \left(1 - \frac{K_o}{K^*}\right)\left(1 - \frac{K_r}{K^*}\right) \qquad (1.1.17)$$

8. The results of a probability analysis carried out by other investigators indicate that maximum rainfall rates at Lake Chapala can be calculated from the following equation

$$r = (25.64\ T_r^{0.19})/T^{0.72} \qquad (1.1.18)$$

where T_r is the return period in years and T is the rainfall duration time in hours.

9. A rainfall contributes oxygen to a body of water in two ways: *a)* by creating turbulence at the air-water interface resulting in an increase in the oxygen transfer coefficient, and *b)* by the direct addition of oxygen contained in the water drops. The relative amounts of oxygen transferred by these two mechanisms can be computed from the following expression

$$p = \frac{1}{1 + \left(\frac{r}{K_r}\right)\left(\frac{S_1}{1 - S_2}\right)} \qquad (1.1.19)$$

In this equation, S_1 and S_2 are the fractional saturated concentrations of oxygen in the drops and in the body of water, respectively. The quantity p represents the fraction of oxygen transferred due to interfacial turbulence, and the quantity $1-p$ represents the fraction transferred by direct addition.

10. The results obtained during the present study may have an additional application in the very serious problems of erosion of soil due to rainfall.

1.2 Recommendations

The present research has been essentially a preliminary study of the effects of wind and rain on surface reaeration. It is recommended that this study be continued along the following lines.

Experimental apparatus should be designed and constructed to measure, separately and in combination, the effects of wind and rain on the oxygen transfer coefficient. It is especially important that additional information be obtained concerning the effect of rain. It is necessary to confirm or modify the assumption that the transfer coefficient is directly proportional to rainfall power.

Theoretical analyses might be carried out to examine various interaction phenomena between wind action and rainfall. For example, the wind surely affects the trajectories and energies of the drops. In turn, the drops probably influence the velocity distribution and roughness heights associated with the wind profile.

Further attention should be given to the subject of drop diameter distributions in artificial and natural rainfall.

It would be desirable to conduct experiments in the field to determine values of surface reaeration coefficients due to natural winds and rainfalls.

2. THE SURFACE REAERATION COEFFICIENT

2.1 Introduction

During recent years there has been growing interest and activity in the development of mathematical models relating to problems of contamination of bodies of water. Such activity, by a large number of engineers and scientists in many countries of the world, has produced the necessary mathematical relationships to provide the means to determine the concentrations and distributions of contaminants in a particular body of water. These relationships are based on the equations of fluid motion and on the equations of mass and heat balance and transfer. As in well-known, numerous mathematical techniques and computing procedures are now available to obtain solutions to these problems.

Clearly, the correctness of these solutions is limited to the correctness of the numerical values of the many coefficients that may be involved in a particular problem. In the classics and very important problem concerning the distributions of biochemical oxygen demand (BOD) and dissolved oxygen (DO) in a river or lake, there are numerous coefficients that must be taken into consideration. For example

- Turbulent diffusion coefficients
- Deoxygenation coefficient
- Transfer coefficient of BOD from bottom deposits
- Transfer coefficient of DO to bottom deposits
- Coefficient of production of DO by photosynthesis
- Coefficient of utilization of DO by respiration
- Surface reaeration coefficient

Although considerable information is now available concerning the numerical values of these coefficients, it is safe to say that much more remains to be learned.

2.2 Purpose of the present study

The present study was devoted to an investigation of the surface reaeration coefficient. As the title of the report indicates, the specific purpose of the study was to determine the effects which wind action and rainfall have on the rate of oxygen transfer at the surface of a body of water.

Other investigators have studied various aspects of the effect of wind on surface transfer; nearly all of the studies along these lines have been concerned with heat transfer. The effect of rainfall on surface transfer has received practically no attention in the past.

Numerous studies have been carried out on the subject of soil erosion due to rainfall. Likewise, considerable research has been conducted on the phenomenon of radar echos from rainstorms. Needless-to-say, specialists working in various fields of meteorology have examined many features of rainfall including the collision and coalescence of raindrops, circulation and oscillation of drops, instability and disruption of drops, freezing mass transfer and evaporation, and various interactions occurring between rainfall and wind structure. Most of these phenomena are of interest to those working in the area of cloud physics; they are not entirely relevant to the subject of the present study. On the other hand, considerable information has been obtained by researchers studying these cloud physics phenomena including information concerning terminal velocities and size distribution of drops in a rainfall. This information has proved to be extremely relevant to the present research.

3. EQUATIONS FOR THE DISTRIBUTIONS OF BOD AND DO

3.1 Introduction

An objective of the present investigation is to obtain information necessary for the computation of the distributions of biochemical oxygen demand (BOD) and dissolved oxygen (DO) in a body of water.

For this reason, the differential equations for the distributions of BOD and DO are presented in the following sections. These equations appear in two-dimensional form for application in a lake or lagoon. Subsequently, they are reduced to one-dimensional form for use in a river or estuary. Steady-state solutions are presented for the one-dimensional problem for the cases in which longitudinal diffusion is included and neglected.

3.2 Equations for the distribution of BOD

Consider an elemental volume of water with dimensions, $H\,dx\,dy$, in which H is the depth and dx and dy are elemental lengths in the horizontal directions, x and y. The velocity components are $u = u(x,y,t)$ and $v = v(x,y,t)$, respectively. The turbulent diffusion coefficients are E_x and E_y.

A mass balance for biochemical oxygen demand in this elemental volume gives the following result

$$\frac{\partial L}{\partial t} + u\frac{\partial L}{\partial x} + v\frac{\partial L}{\partial y} + K_1 L = \frac{\partial}{\partial x}\left(E_x\frac{\partial L}{\partial x}\right) + \frac{\partial}{\partial y}\left(E_y\frac{\partial L}{\partial y}\right) + L^* \qquad (3.2.1)$$

in which L is the concentration of BOD, K_1 is the deoxygenation coefficient and

$$L^* = \frac{1}{H}(M_r + M_b) \qquad (3.2.2)$$

where M_r and M_b are the amounts of BOD entering the elemental volume per unit time and area from rainfall and from bottom deposits, respectively.

The one-dimensional form of eq. *3.2.1* is

$$\frac{\partial L}{\partial t} + u\frac{\partial L}{\partial x} + K_1 L = \frac{\partial}{\partial x}\left(E_x\frac{\partial L}{\partial x}\right) + L^* \qquad (3.2.3)$$

If there is no change in the concentration of BOD with respect to time ($\partial L/\partial t = 0$), if $u = U =$ constant and if $E_x = E_o =$ constant, the solution to eq. *3.2.3* is

$$L = L_o e^{J_1 x} + \frac{L^*}{K_1}(1 - e^{J_1 x}) \qquad (3.2.4)$$

where L_o is the concentration at $x = 0$ and

$$J_1 = \frac{U}{2E_o}\left[1 - \sqrt{1 + \frac{4K_1 E_o}{U^2}}\right] \qquad (3.2.5)$$

If diffusion in the x-direction is neglected ($E_o = 0$), then $J_1 = -K_1/U$ and eq. *3.2.4* becomes

$$L = L_o e^{-K_1 x/U} + \frac{L^*}{K_1}(1 - e^{-K_1 x/U}) \qquad (3.2.6)$$

From this equation it is observed that the concentration of BOD for large values of x is $L_\infty = L^*/K_1$.

3.3 Equations for the distribution of DO

A similar mass balance for dissolved oxygen in the elemental volume gives the following equation

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + K_1 L + K_2 C = \frac{\partial}{\partial x}\left(E_x \frac{\partial C}{\partial x}\right) + \frac{\partial}{\partial y}\left(E_y \frac{\partial C}{\partial y}\right) + C^* \qquad (3.3.1)$$

in which C is the concentration of DO, K_2 is the reaeration coefficient and

$$C^* = \frac{1}{H}(N_r - N_b) + K_2 C_s + K_p - K_r \qquad (3.3.2)$$

where N_r is the amount of DO entering the elemental volume per unit time and area from rainfall and N_b is the amount of DO leaving the elemental volume to bottom deposits. The equilibrium concentration of oxygen is C_s; values of C_s are listed in the Table A.1 of the appendix. The amount of oxygen produced by photosynthesis per unit time and volume is K_p and the amount of oxygen utilized by respiration per unit time and volume is K_r.

The one-dimensional form of eq. *3.3.1* is

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + K_1 L + K_2 C = \frac{\partial}{\partial x}\left(E_x \frac{\partial C}{\partial x}\right) + C^* \qquad (3.3.3)$$

If there is no change in the concentration of DO with respect to time $(\partial C/\partial t = 0)$, if $u = U =$ constant and if $E_x = E_o =$ constant, the solution to eq. *3.3.3* is

$$C = C_o e^{J_2 x} + \frac{(C^* - L^*)}{K_2}(1 - e^{J_2 x}) - \frac{(K_1 L_o - L^*)}{(K_2 - K_1)}(e^{J_1 x} - e^{J_2 x}) \qquad (3.3.4)$$

where C_o is the concentration at $x = 0$ and

$$J_2 = \frac{U}{2E_o}\left[1 - \sqrt{1 + \frac{4K_2 E_o}{U^2}}\right] \qquad (3.3.5)$$

If diffusion in the x-direction is neglected $(E_o = 0)$, then $J_2 = -K_2/U$ and eq. *3.3.4* becomes

$$C = C_o e^{-K_2 x/U} + \frac{(C^* - L^*)}{K_2}(1 - e^{-K_2 x/U}) - \frac{(K_1 L_o - L^*)}{(K_2 - K_1)}(e^{-K_1 x/U} - e^{-K_2 x/U}) \qquad (3.3.6)$$

It is observed in this equation that the concentration of DO for large values of x is $C_\infty = (C^* - L^*)/K_2$. The location of the point of minimum concentration of DO is obtained from eq. *3.3.4*. The result is

$$x_c = \frac{1}{(J_1 - J_2)} \log_e \frac{J_2}{J_1} \left[1 + \frac{(K_2 - K_1)}{(K_1 L_o - L^*)} \left\{ C_o - \frac{(C^* - L^*)}{K_2} \right\} \right] \tag{3.3.7}$$

The value of the minimum concentration of DO can be obtained by substituting the value of x_c into eq. *3.3.4.* If longitudinal diffusion is neglected $(E_o = 0)$, the location of the point of minimum concentration is

$$x_c = \frac{U}{(K_2 - K_1)} \log_e \frac{K_2}{K_1} \left[1 + \frac{(K_2 - K_1)}{(K_1 L_o - L^*)} \left\{ C_o - \frac{(C^* - L^*)}{K_2} \right\} \right] \tag{3.3.8}$$

The function, J_1 and J_2 defined in eqs. *3.2.5* and *3.3.5*, are presented in graphical from in fig 3.1. The solutions given by eqs. *3.2.4* and *3.3.4* were obtained by O'Connor (36) in a slightly less generalized form. The solution expressed by eqs. *3.2.6* and *3.3.6* are the well-known equations of Streeter-Phelps (43) for the longitudinal distributions of BOD an DO in a river.

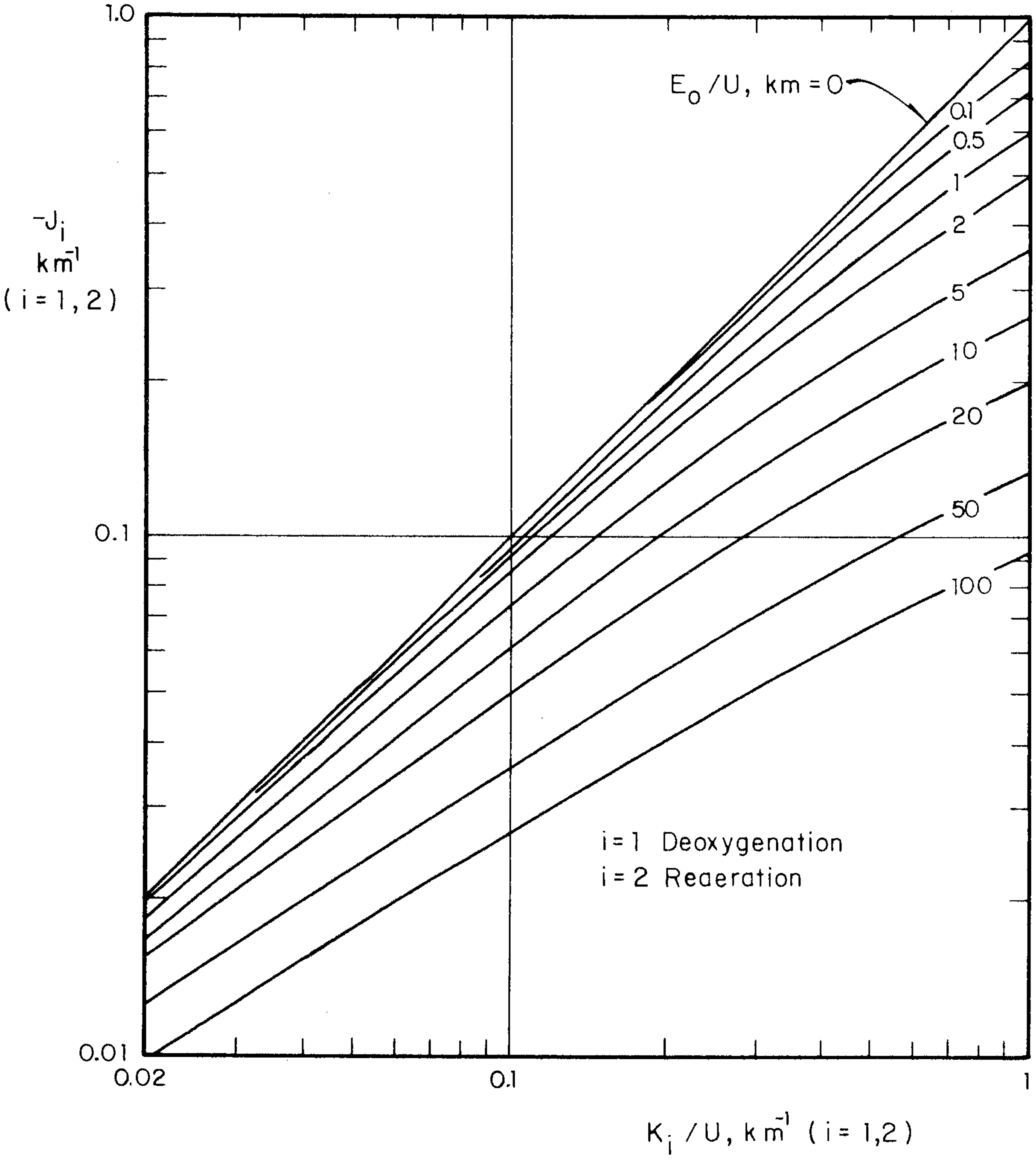

Fig 3.1. Plot of the functions, J_1 and J_2

4. EFFECT OF WIND ON SURFACE REAERATION

4.1 Introduction

The atmosphere is the most important source of oxygen for a body of water. The rate at which oxygen is transferred across an air-water interface depends on the magnitude of the surface reaeration coeficient. Numerous studies have shown that the value of this coefficient, in a river or estuary, depends on the average velocity and depth of flow.

In a lake or lagoon, however, there is generally no well-defined velocity of flow corresponding to that in a river. A recent analysis of oxygen and heat transfer data indicates that wind action is the most important factor in establishing the magnitude of the reaeration coefficient in lakes and lagoons. This topic is considered in the following sections.

4.2 The reaeration coefficient in rivers and estuaries

The oxygen transfer coefficient, K, has its definition in the following equation

$$M = K A (C_s - C) \qquad (4.2.1)$$

in which M is the rate of oxygen transfer across an air-water interface of area A, C_s is the equilibrium concentration of oxygen and C is the concentration of oxygen in a well-mixed column of water of depth H. The units of the oxygen transfer coefficient, sometimes identified as the exit coefficient, are those of a velocity.

The surface reaeration coefficient, K_2, is defined as follows

$$N = K_2 A H (C_s - C) \tag{4.2.2}$$

From eqs. *4.2.1* and *4.2.2*, it is clear that

$$K = K_2 H \tag{4.2.3}$$

The unit of K_2 is a reciprocal time.

Over the years, many investigators have studied the phenomenon of surface reaeration in rivers and estuaries. The classic work of O'Connor and Dobbins (35) has been extended by Churchill, Elmore and Buckingham (10), Krenkel and Orlob (25), Owens, Edwards and Gibbs (37), Tsivoglou (45,46) and numerous other researchers. A comprehensive summary of information on surface reaeration in natural streams has been published by Kramer (24).

Most of these studies have produced empirical formulas of the type

$$K_2 = a \frac{V^m}{H^n} \tag{4.2.4}$$

where V and H are the average velocity and depth of flow, respectively, and a, m and n are constants. Although there are wide variations in the results of these studies, the fact is that a large amount of data now exists on the subject of reaeration in natural streams.

4.3 The reaeration coefficient in lakes and lagoons

In contrast to the situation concerning rivers and estuaries, not much information is available regarding surface reaeration in lakes and lagoons. The most frequently cited work is that of Downing and Truesdale (12) who carried out a laboratory study to determine the effect of wind on oxygen transfer at an air-water interface. Brady, Graves and Geyer (8) developed an empirical equation to compute the surface heat transfer coefficient as a function of the wind velocity and Hindley and Miner (20) conducted field experiments along these lines.

In a recent publication, Banks (1) analyzed the results of these earlier investigations. He developed the correlation shown in fig 4.1 between the wind velocity, U, and the transfer coefficient, K_o; the subscript, o, refers to transfer due to wind action. The oxygen transfer data shown in fig 4.1 are those of Downing and Truesdale (12); the heat transfer data were obtained by Hindley and Miner (20). These two types of transfer are compared on the basis of the following analogy.

The equation for the one-dimensional distribution of dissolved oxygen is given by eq. *3.3.3*. Setting $K_1 = 0$ and $C^* = K_2 C_s$ eq. *3.3.3* becomes

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} = \frac{\partial}{\partial x}\left(E_x \frac{\partial C}{\partial x}\right) + K_2 (C_s - C) \tag{4.3.1}$$

Echâvez (15) has shown that the equation for the one-dimensional distribution of temperature is

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = \frac{\partial}{\partial x}\left(E_x \frac{\partial T}{\partial x}\right) + \frac{k}{\rho\, C_v\, H}(T_e - T) \quad (4.3.2)$$

where T is the temperature, T_e is the equilibrium temperature, k is the surface heat transfer coefficient, ρ is the density and C_v is the specific heat. A comparison of eqs. *4.3.1* and *4.3.2* shows that the quantity, $k/\rho C_v H$, in the temperature distribution problem is analogous to the quantity, K_2, in the oxygen distribution problem. Alternatively, from eq. *4.2.3*, the oxygen transfer coefficient, K, is equivalent to the quantity, $k/\rho C_v$. Thus, the ordinates of the heat transfer data presented in fig. 4.1 are values of $k/\rho C_v$.

The equation of the line shown in fig. 4.1 is

$$K_o = 10^{-6}\left[8.43\sqrt{U} - 3.67\ U + 0.43\ U^2\right] \quad (4.3.3)$$

In this equation, the units of K_o and U are m/s. The velocity of the wind, U, is based on a reference elevation of 10 m.

A numerical example is presented. Suppose that a wind with velocity, $U = 35$ km/h = 9.7 m/s, blows steadily over a lake whose average depth is $H = 10$ m. From fig 4.1 or eq. *4.3.3* the value of the oxygen transfer coefficient is $K_o = 3.1 \times 10^{-5}$ m/s. From eq. *4.2.3*, the value of the reaeration coefficient is

$$K_{o,2} = (3.1 \times 10^{-5})/10 = 3.1 \times 10^{-6}\ s^{-1} = 0.27\, d^{-1}$$

From eq. *4.2.3* and *4.3.3*, the following expression is obtained for the reaeration coefficient, $K_{o,2}$

$$K_{o,2} = \frac{1}{H}\left[0.384\sqrt{U} - 0.088\ U + 0.0029\ U^2\right] \quad (4.3.4)$$

in which the various quantities have the following units:

Reaeration coefficient, $K_{o,2}$:	d^{-1}
Wind velocity, U:	km/h
Depth, H:	m

It is observed that eq. *4.3.4* is "similar" to the expression for the reaeration coefficient for rivers given by eq. *4.2.4*.

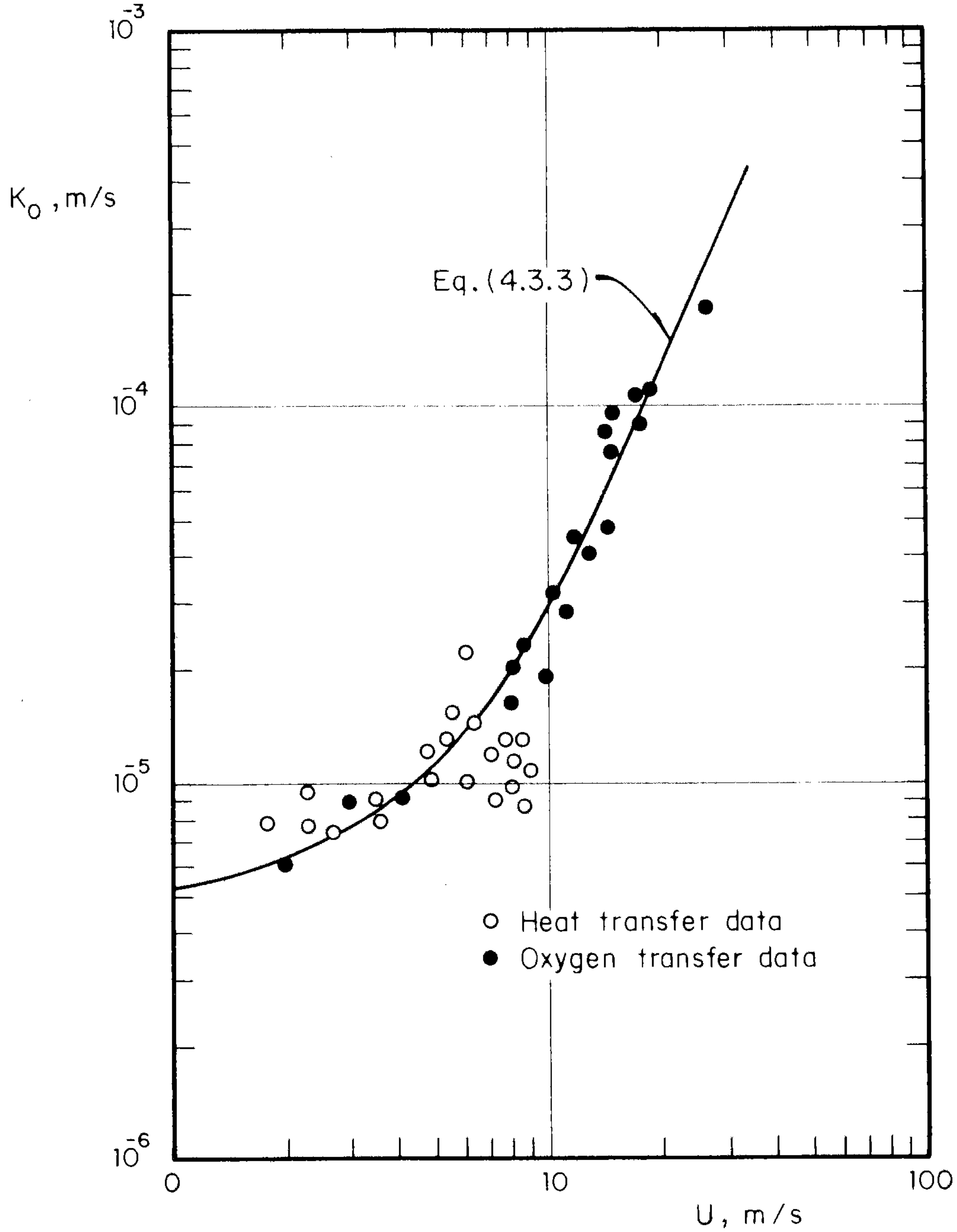

Fig 4.1. *Oxygen transfer coefficient as a function of the wind velocity*

5. VELOCITY OF FALL OF WATER DROPS IN AIR

5.1 Introduction

In order to calculate the kinetic energy of a drop of rain and the energy *flux* of a rainfall, it is necessary to know the size of the drops and their corresponding fall velocities. The following sections are devoted to this subject.

Various equations of the Stokes-type are listed even though these equations are valid only for small Reynolds numbers. Except for the very small drops associated with drizzle and mist, the diameters and velocities of drops in a typical rainfall are sufficiently large to yield Reynolds numbers beyond the range of validity of Stokes law. Consequently, the fall velocities of drops of relatively large diameters, corresponding to large Reynolds numbers, are examined in detail. The influence of reduced air density at high elevation on drop velocities is considered.

5.2 Equation for the velocity of a spherical particle

Consider a spherical particle whose diameter is D, density is ρ' and viscosity is μ' falling at constant velocity V, through a fluid whose density is ρ and viscosity is μ. A dimensional analysis yields the result that the force which the fluid exerts on the particle is

$$F = \frac{1}{2} \rho \, C_d \, \frac{\Pi}{4} \, D^2 \, V^2 \tag{5.2.1}$$

where C_d is a drag coefficient whose value depends on the Reynolds number, i.e.,

$$C_d = C_d\,(Re); \quad Re = \rho\, V\, D/\mu \tag{5.2.2}$$

The summation of the forces acting on the particle is equal to zero. Therefore

$$g\,\frac{\Pi}{6} D^3\,(\rho' - \rho) = \frac{1}{2}\rho\, C_d\,\frac{\Pi}{4} D^2\; V^2 \tag{5.2.3}$$

This relationship yields the desired equation for the velocity

$$V = \sqrt{\frac{4}{3}\;\frac{g(\rho' - \rho)\,D}{\rho\; C_d}} \tag{5.2.4}$$

5.3 Velocity corresponding to small Reynolds numbers

If the particle is a solid sphere with a Reynolds number less than approximately 1.0, the force which the fluid exerts on the particle is

$$F = 3\,\Pi\,\mu\; V\, D \tag{5.3.1}$$

This is the classic result of Stokes. Substituting eq. *5.3.1* into eq. *5.2.1* gives

$$C_d = 24/Re \tag{5.3.2}$$

Employing eq. *5.3.2* in eq. *5.2.4* yields

$$V = \frac{g\,(\rho' - \rho)\,D^2}{18\,\mu} \tag{5.3.3}$$

This result, generally identified as Stokes law, gives the velocity of a solid sphere in a viscous fluid for low values of the Reynolds number.

Over a period of many years, numerous investigators have developed modifications of Stokes law. Some of the results of these investigations are given below. They are presented as modifications of the drag force, F. The corresponding expressions for the velocity, V, can be determined from the equation

$$g\,\frac{\Pi}{6}\; D^3\;(\rho' - \rho) = F \tag{5.3.4}$$

1. *Stokes* *Ref:* Dryden *et al* (13)

$F = 3\,\Pi\mu\;V\,D$
$C_d = 24/Re$
$V = g\,(\rho' - \rho)\,D^2\;/\;18\,\mu$

Note: Valid for solid sphere for $Re < 1.0$.

2. *Stokes-Oseen* *Ref:* Landau and Lifshitz (27)

$$F = 3 \Pi \mu V D \left[1 + \frac{3}{16} Re\right]$$

Note: Valid for solid sphere for $Re < 2.0$

3. *Stokes-Cunningham* *Ref:* Dryden *et al* (13)

$$F = 3 \Pi \mu V D \left[\frac{1}{1 + A(\lambda/D)}\right]$$

where $\lambda = \mu/0.350\ \rho\ v_m$

v_m = average molecular velocity

A = 1.4 to 2.0

Note: Valid for solid sphere when the diameter, D, is of the same order of magnitude as the mean free path, λ, of a molecule of the fluid.

4. *Stokes-Millikan* *Ref.* Dryden *et al* (13)

$$F = 3 \Pi \mu V D \left[\frac{1}{1 + 1.728(\lambda/D) + 0.290 \exp(-0.63 D/\lambda)}\right]$$

where $\lambda = \mu/0.350\ \rho\ v_m$

v_m = average molecular velocity

Note: Valid for solid spheres when the diameter, D, is of the same order of magnitude as the mean free path, λ, of a molecule of the fluid. A refinement of the Stokes-Cunningham equation.

5. *Stokes-Basset* *Ref:* Lamb (26)

$$F = 3 \Pi \mu V D \left[\frac{f\alpha + 2\mu}{f\alpha + 3\mu}\right]$$

where f = coefficient of sliding friction
if $f = \infty$, $F = 3 \Pi \mu V D$ (solid sphere)
if $f = 0$, $F = 2 \Pi \mu V D$ (gas sphere)

Note: Valid for solid or fluid spheres for small Re.

6. *Rybczynski-Hadamard* *Ref:* Landau and Lifshitz (27)

$$F = 3 \Pi \mu V D \left[\frac{2\mu + 3\mu'}{3(\mu + \mu')}\right]$$

if $\mu' = \infty$ (solid sphere), $F = 3\,\Pi\,\mu\,V\,D$
$\mu' = 0$ (gas sphere), $F = 2\,\Pi\,\mu\,V\,D$

Note: Valid for solid or fluid spheres for small *Re*

7. *Boussinesq* *Ref:* Dryden *et al* (13)

$$F = 3\,\Pi\,\mu\,V\,D \left[\frac{f + (2\,\mu + 3\,\mu')(D/2)}{f + 3\,(\mu + \mu')\,(D/2)} \right]$$

where f = coefficient of sliding friction
if $f = 0$, reduces to Rybczynski-Hadamard

Note: Valid for solid or fluid spheres for small *Re.*

It should be pointed out that in a typical rainfall, the velocity and diameter of the water drops are usually sufficiently large to produce Reynolds numbers greater than the range in which Stokes law is valid. Rainfalls consisting of mist or drizzle (gentle and steady fall of very small droplets) may correspond to Reynolds number within the Stokes range. However, the energy flux associated with this type of rainfall is very small compared to rainfalls composed of larger drops. Accordingly, the above equations, corresponding to small Reynolds numbers are included mostly for the sake of completness.

5.4 *Velocity corresponding to large Reynolds numbers*

When the Reynolds number is larger than about 1.0, the velocity of a spherical particle in a fluid is no longer described by Stokes law. For a drop of water falling in air, this critical value, $Re = 1.0$, corresponds to a diameter $D = 0.08$ mm, and a velocity, $V = 19$ cm/s.

Experimental studies to determine the velocity of water drops in air have been carried out by numerous investigators over a period of many years. One of the earliest studies was that of Lenard (30). His results were confirmed fairly well some years later by Laws (28). The results of these two studies are summarized in table 5.1.

TABLE 5.1 VELOCITY OF RAINDROPS, *V*, AS A FUNCTION OF DIAMETER, *D*

	Velocity, m/s	
Diameter, mm	Lenard (30)	Laws (28)
1.0	4.4	––
2.0	5.9	6.9
3.0	7.0	8.0
4.0	7.7	8.8
5.0	7.9	9.2
5.5	8.0	9.3
6.5	7.8	––

Subsequently, other investigators considered this subject. In chronological order these included the studies of Spilhaus (42), Gunn and Kinzer (18), Best (5), Blanchard (7), Beard and Pruppacher (4), Foote and Du Toit (17) Wobus, Murray and Koenig (48), Dingle and Lee (11), and Lin and Lee (31).

The following regression equation was developed by Dingle and Lee (11) relating velocity, V (cm/s), to diameter, D (mm), over the range of diameters from 0.1 to 5.8 mm.

$$V = 5.49\,D^3 - 88.80\,D^2 + 491.84\,D - 16.60 \qquad \textbf{(5.4.1)}$$

In Table 5.2 the results computed from this equation are compared with the results obtained by Gunn and Kinzer (18) and by Wobus, Murray and Koenig (48). It is observed that there is close agreement among the three sets of values. The average velocities and corresponding values of the Reynolds numbers, Re, and drag coefficients, C_d, are shown in Table 5.2.

The relationship, $C_d = C_d(Re)$, is presented in fig 5.1. It is seen that the drag coefficient for both solid and liquid spheres is accurately predicted by Stokes law, $C_d = 24/Re$, for values of Reynolds number less than about 1.0. It is also observed in fig 5.1 that the drag coefficient for a solid sphere is slightly larger than the drag coefficient fro a liquid drop for values of Re less than approximately 500. For values of Re greater than 1000, the drag coefficient of a liquid drop increases rather sharply. The diameter of a drop is defined as the diameter of a sphere with the same volume as the actual drop.

Lin and Lee (31) obtained the following expression for the drag coefficient of a sphere for Reynolds numbers less than 1000.

$$C_d = \frac{24}{Re}\left[1 + 0.2207\sqrt{Re} + 0.0125\,Re\right] \qquad \textbf{(5.4.2)}$$

The velocities of water drops presented in table 5.2 correspond to sea-level conditions $(z = 0)$. It is observed, in eq. *5.2.4*, that the velocity of a drop of given diameter depends on the density of the air, ρ, and the drag coefficient, C_d, in the following manner.

$$V = \frac{constant}{\sqrt{\rho\, C_d}} \qquad \textbf{(5.4.3)}$$

If C_d were constant, the velocity would be inversely proportional to the square root of the density of the air. However, as mentioned, the drag coefficient is not constant. Accordingly, the following relationship, developed by Foote and Du Toit (17), was employed to determine the increase of velocity with elevation.

$$\frac{V}{V_o} = 10^Y\left[1 + 0.0023\,(1.1 - \frac{\rho}{\rho_o})\,(T_o - T)\right] \qquad \textbf{(5.4.4)}$$

where

$$Y = 0.43\,\log_{10}\left(\frac{\rho_o}{\rho}\right) - 0.4\left[\log_{10}\,\frac{\rho_o}{\rho}\right]^{2.5} \qquad \textbf{(5.4.5)}$$

TABLE 5.2. VALUES OF DROP DIAMETER, VELOCITY, REYNOLDS NUMBER AND DRAG COEFFICIENT AT SEA–LEVEL

Diameter, mm	Velocity, cm/s (1)	(2)	(3)	Average	Reynolds number	Drag coefficient
0.1	27	32	27	27	1.8	14.62
0.2	72	78	72	74	9.9	3.89
0.3	117	123	117	119	23.8	2.26
0.4	162	166	162	163	43.5	1.60
0.5	206	208	206	207	69.0	1.24
0.6	247	248	247	247	98.8	1.05
0.7	287	286	287	287	134	0.91
0.8	327	323	327	326	174	0.80
0.9	367	358	367	364	219	0.72
1.0	403	392	403	399	266	0.67
1.2	464	455	464	461	369	0.60
1.4	517	513	516	515	481	0.56
1.6	565	566	565	565	603	0.53
1.8	609	613	610	611	734	0.51
2.0	649	656	652	652	870	0.50
2.2	690	694	690	691	1014	0.49
2.4	727	728	724	726	1162	0.49
2.6	757	758	755	757	1313	0.48
2.8	782	785	782	783	1462	0.49
3.0	806	808	806	807	1615	0.49
3.2	826	828	827	827	1765	0.50
3.4	844	845	845	845	1916	0.51
3.6	860	859	860	860	2065	0.52
3.8	872	871	873	872	2210	0.53
4.0	883	881	883	882	2353	0.55
4.2	892	889	891	891	2496	0.56
4.4	898	896	897	897	2633	0.58
4.6	903	901	902	902	2768	0.60
4.8	907	905	906	906	2901	0.62
5.0	909	909	909	909	3032	0.64
5.2	912	912	912	912	3163	0.67
5.4	914	914	914	914	3292	0.69
5.6	916	917	915	916	3421	0.71
5.8	917	920	916	918	3551	0.74

Column (1): Gunn and Kinzer (18); (2): Dingle and Lee (11); (3): Wobus, Murray and Koenig (48)

The subscripts refer to sea-level conditions. The standard atmosphere shown in Table A.2 of the appendix was employed for the computations. The results are presented in Table 5.3 and in fig 5.2.

It is observed that a drop with diameter, $D = 6.0$ mm, has a velocity, $V = 918$ cm/s, at sea-level, $z = 0$. The same drop has a velocity, $V = 1{,}185$ cm/s, at an elevation $z = 5{,}000$ m. As discussed in sec. 7.6, the energy of a spherical particle is

$$E = \frac{10^{-1}}{2} \rho' \frac{\Pi}{6} D^3 V^2 \tag{5.4.6}$$

From this equation, the energy of a drop of 6.0 mm diameter at sea level is $E_o = 4{,}765.5\ \mu J$. At an elevation of 5,000 m, the energy of the same drop is $E = 7{,}940.7\ \mu J$. Clearly a drop striking the ground or a body of water at a high elevation has considerably more energy than does the same drop striking the ground or a body of water at sea-level. This indicates that a rainfall occuring over an area at high elevation possesses greater power for the erosion of soil or the reaeration of a body of water than does the same rainfall at sea-level.

5.5 Maximum velocity and size of water drops

Commencing with the early studies of Lenard (30), numerous investigators have established that a drop of water falling in air acquires a maximum diameter and a maximum velocity. At sea-level conditions, these maximum values are probably no larger than $V = 920$ cm/s and $D = 6.0$ mm. Greater values are prevented because of the instability of the drop. On this subject, Richardson (39) indicates

> "A drop of liquid in motion through another fluid differs in its behavior from a solid sphere in that it may *a)* be deformed, *b)* have a circulation set up within itself by the shear effect of the relative motion of the two fluids. These effects upset the stability of the drop, causing it to oscillate about the spherical shape and eventually to burst into fragments or at least into smaller drops."

Richardson proposed a Bond number, *Bo*, to describe this phenomenon

$$Bo = \frac{g(\rho' - \rho) D^2}{\sigma} \tag{5.5.1}$$

where σ is the coefficient of interfacial tension (μN/cm). From this equation, it is observed that the Bond number is essentially a ratio of the net weight of the particle, $g(\rho' - \rho)D^3$, to a force due to surface tension, σD, Richardson reported that the particle begins to deform from its spherical shape when *Bo* exceeds about 0.4 and that internal circulation within the drop becomes significant when *Bo* is greater than about 1.5. His experiments indicated that a water drop falling in air is fragmented into smaller droplets when *Bo* is approximately 10. This value is somewhat larger than the calculated value, corresponding to $D = 6.0$ mm

$$Bo = \frac{g(\rho' - \rho) D^2}{\sigma} = \frac{980\,(1.0 - 0)\,(0.6)^2}{730} = 4.6 \tag{5.5.2}$$

TABLE 5.3 VALUES OF DROP DIAMETER VS. VELOCITY FOR VARIOUS ELEVATION IN METERS ABOVE SEA–LEVEL

Diameter, mm	Velocity (cm/s) at elevation, z (m), above sea-level					
	0	1000	2000	3000	4000	5000
0.1	27	28	30	31	33	35
0.2	72	75	79	83	87	93
0.3	117	122	128	134	141	151
0.4	162	169	177	186	196	209
0.5	206	214	226	238	251	266
0.6	247	256	269	283	299	317
0.7	287	298	312	328	347	368
0.8	327	341	355	373	395	419
0.9	367	384	398	419	443	470
1.0	403	421	442	465	491	520
1.2	464	485	505	531	564	594
1.4	517	540	567	597	637	667
1.6	565	591	615	648	688	724
1.8	609	637	663	699	739	781
2.0	649	679	712	750	791	838
2.2	690	722	755	795	839	889
2.4	727	760	798	840	887	939
2.6	757	792	826	870	919	973
2.8	782	818	855	900	951	1007
3.0	806	843	884	931	983	1041
3.2	826	864	905	953	1006	1066
3.4	844	883	926	975	1029	1090
3.6	860	896	940	990	1045	1107
3.8	872	909	954	1005	1061	1124
4.0	883	923	969	1020	1077	1140
4.2	892	926	977	1029	1086	1150
4.4	898	939	985	1037	1095	1159
4.6	903	943	989	1041	1099	1164
4.8	907	947	993	1045	1103	1169
5.0	909	951	997	1050	1108	1174
5.2	912	954	1000	1053	1112	1177
5.4	914	956	1003	1056	1115	1180
5.6	916	958	1005	1058	1117	1182
5.8	917	959	1006	1059	1118	1184

A bond number, $Bo = 10$, corresponds to a diameter of approximately 8.6 mm.

As indicated, a drop begins to deform from its spherical shape when the Bond number is about 0.4. Its subsequent shape depends on the density ratio, $\rho^* = \rho'/\rho$, and viscosity ratio, $\mu^* = \mu'/\mu$. Dryden *et al* (13) have presented the results of an analysis carried out years ago by Saito. A quantity, f, is defined as follows

$$f = f_1 - f_2 = \rho^* \left[1 + \mu^*\right] - 10\left[1 + \frac{319}{100}(\mu^*) + \frac{111}{50}(\mu^*)^2 + \frac{3}{200}(\mu^*)^3\right] \quad (5.5.3)$$

Saito showed that the sphere is deformed into a prolate ellipsoid or an oblate ellipsoid depending on wheter f is greater or less than zero. To illustrate the application of eq. *5.5.3*, computations are presented in Table 5.4 for the following fluid combinations: *a)* water drop in air and *b)* mercury drop in air.

TABLE 5.4 COMPARISON OF THE SHAPES OF A WATER DROP AND A MERCURY DROP FALLING IN AIR (SAITO'S CRITERION)

Quantity	Units	Water drop in air (20 Celsius)	Mercury drop in air (20 Celsius)
ρ'	g/cm^3	1.00	13.55
ρ	g/cm^3	1.226×10^{-3}	1.226×10^{-3}
μ'	dPa–s	1.00×10^{-2}	1.60×10^{-2}
μ	dPa–s	1.78×10^{-4}	1.78×10^{-4}
$\rho^* = \rho'/\rho$		0.82×10^3	11.05×10^3
$\mu^* = \mu'/\mu$		0.56×10^2	0.90×10^2
f_1		4.67×10^4	100.6×10^4
f_2		9.78×10^4	29.2×10^4
$f = f_1 - f_2$		-5.11×10^4	$+71.4 \times 10^4$
Shape		oblate	prolate

The results given in Table 5.4 indicate that a drop of water in air is initially deformed into an oblate ellipsoid whereas a drop of mercury in air is deformed into a prolate ellipsoid.

Based on the critical value of the Bond number, $Bo = 0.4$, deformation from a spherical shape begins to occur when the diameter is approximately 1.7 mm. This value corresponds to a velocity of about 590 cm/s and a Reynolds number of 690. The diameter of a drop deformed considerably from a spherical shape is defined as the diameter of a sphere with the same volume as the actual drop.

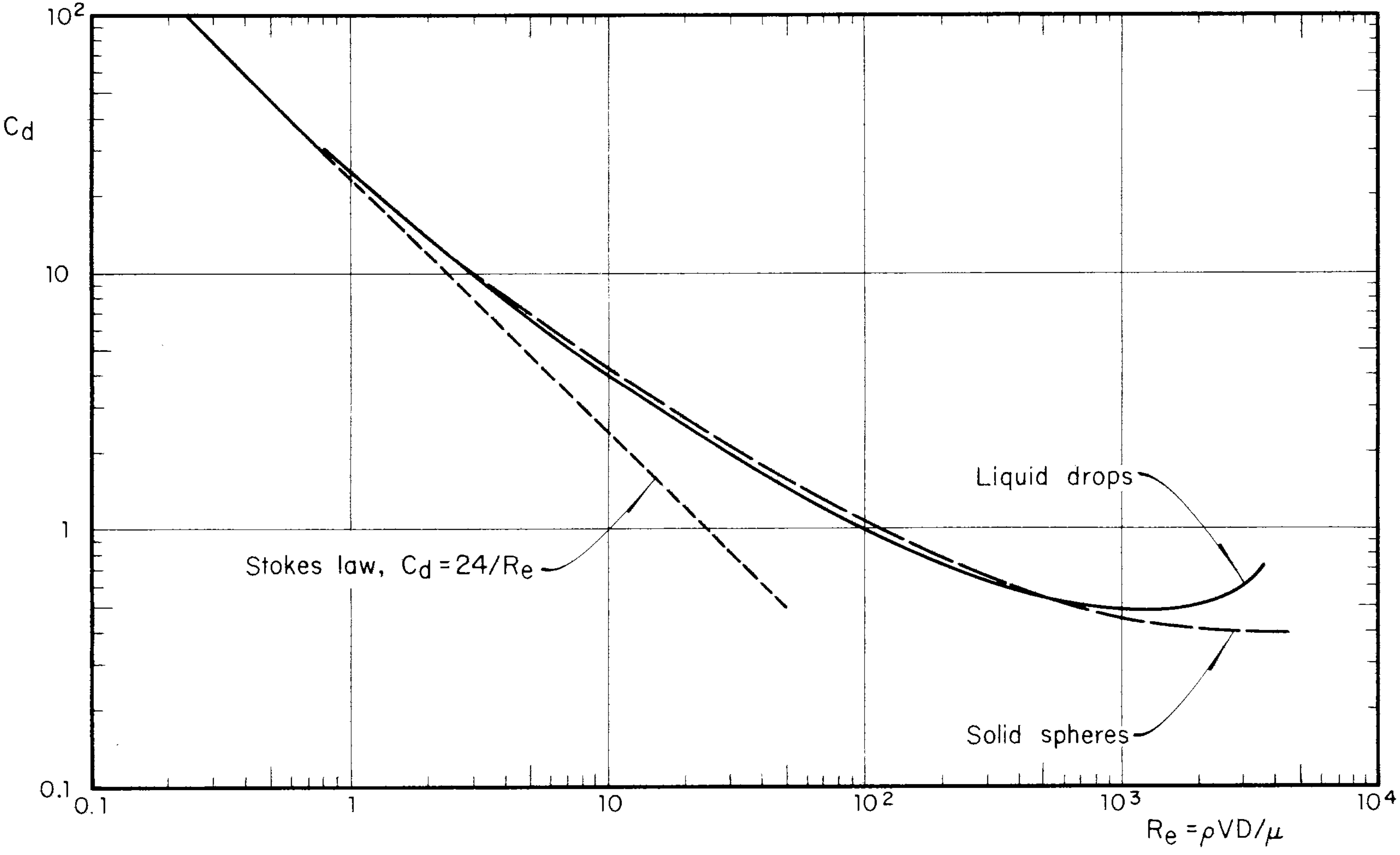

Fig 5.1. The drag coefficient of a sphere as a function of the Reynolds number

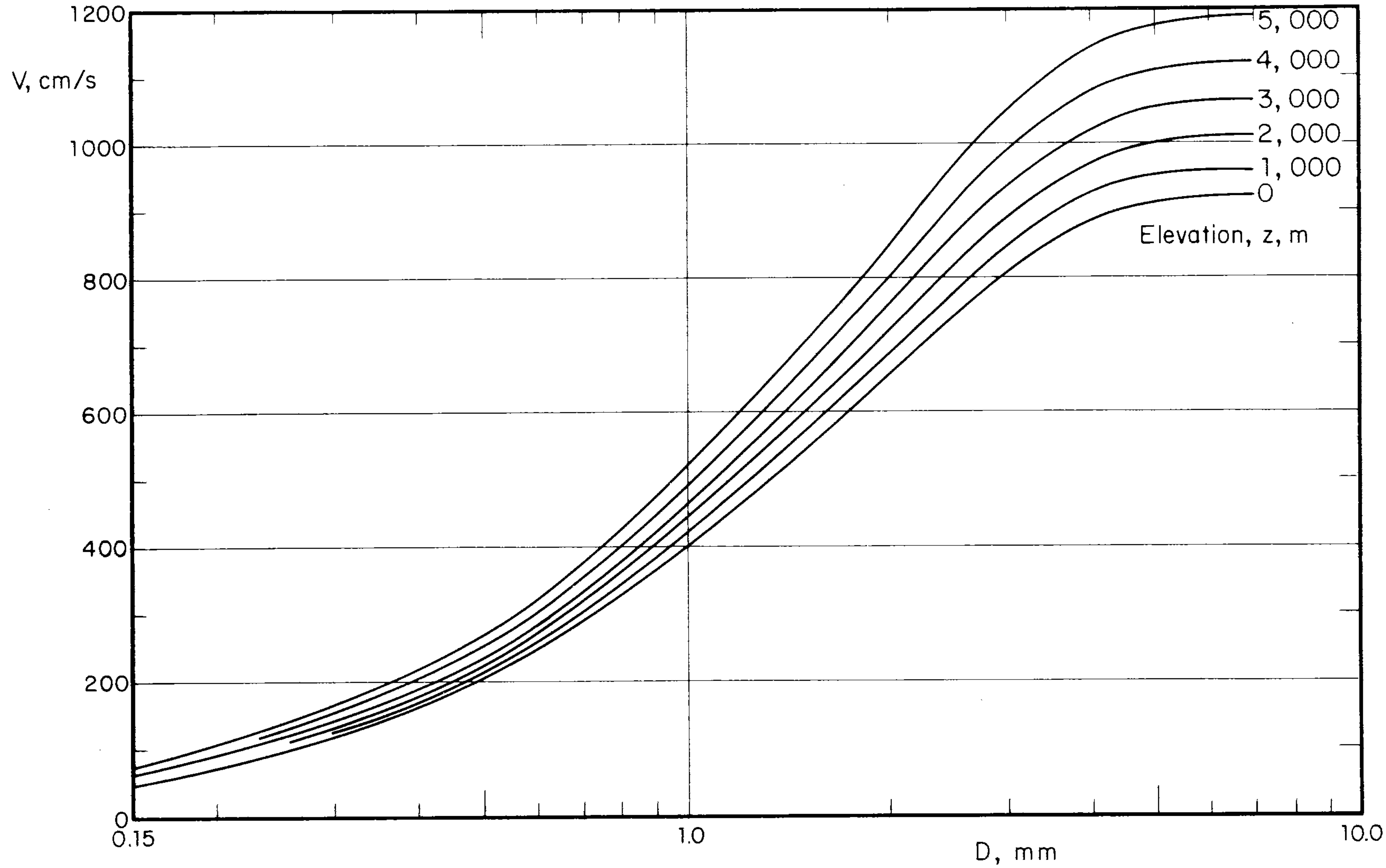

Fig 5.2. *Velocities of water drops falling in air as a function of drop diameter and elevation above sea-level*

6. DISTRIBUTION OF SIZES OF WATER DROPS IN A RAINFALL

6.1 *Introduction*

To determine the energy flux or power of a rainfall it is necessary to know the velocity of fall of the drops. This subject was examined in chapter 5. However, in any rainfall there is a wide range of sizes, and hence fall velocities, of the drops. Accordingly, it is necessary to have information concerning the distribution of sizes of drops in a rainfall.

As indicated in the following sections, the results of two experimental studies conducted some years ago provide the desired information. The distribution of sizes is exponential in character; the precise distribution is strongly influenced by the rate of rainfall.

6.2 *Size distribution of raindrops*

The subject of size distribution of drops in a rainfall was considered in experimental studies conducted by Laws and Parsons (29) and by Marshall and Palmer (32).

The laboratory studies of Laws and Parsons involved simulated rainfalls at the following rates: $r = 0.254$, 1.27, 2.54, 12.70, 25.4, 50.8, 101.6 and 152.4 mm/h. The results of four of these series of experiments (0.254, 2.54, 25.4 and 152.4 mm/h) are presented in fig 6.1. The abscissa of this plot is the diameter, D, of a drop in cm. The ordinate of the plot is the quantity, N^*. The product $N^* \Delta D$, is the number of drops per m^3 with diameters between D and $D + \Delta D$ in mm. For example, with $\Delta D = 0.30$ mm and $r = 25.4$ mm/h, the number of drops with diameters in the range from 0.185 to 0.215 cm (average diameter, $D = 0.20$ cm) is 125 (from fig 6.1) x 0.30 = 38 per m^3.

Similar experiments were conducted by Marshall and Palmer with the following rainfall rates: 1.0, 2.8, 6.3 and 23.0 mm/h. Their results are presented in fig 6.2. The solid lines shown in figs 6.1 and 6.2 were computed from the generalized distribution equation presented in the following section.

6.3 Generalized distribution of raindrop diameters

The abscissa, D (cm), of figs 6.1 and 6.2 were modified by dividing by the quantity $r^{0.21}$, where r is the rainfall rate in mm/h. The resulting correlation between N and $D/r^{0.21}$ for the data of Laws and Parsons is presented in fig 6.3. The correlation corresponding to the results of Marshall and Palmer is shown in fig 6.4..

The line shown in figs 6.3 and 6.4 is the same for the two sets of data. The equation of this line is

$$N^* = N_o \ exp\,(-\lambda D) \qquad (6.3.1)$$

in which $N_o = 8{,}000\ m^{-3}\ mm^{-1}$ and

$$\lambda = 41.0/r^{\,0.21} \qquad (6.3.2)$$

where D is in cm and r is in mm/h. The solid lines of figs 6.1 and 6.2 were computed from eqs. *6.3.1* and *6.3.2.*

Figs 6.3 and 6.4 provide the generalized relationships for determining the distribution of diameters of drops in a given rainfall. Both plots display very good correlation and the close agreement between the results of the two separate investigations is remarkable. In the experiments by Laws and Parsons it is observed in fig 6.3 that measured values of N^* are somewhat less than computed values in the range of $D/r^{\,0.21}$ from about 0.03 to 0.10.

The numerical example given in Table 6.1 is presented to illustrate the method of computation.

TABLE 6.1. EXAMPLE OF THE METHOD OF COMPUTATION

Rainfall rate, r = 25 mm/h; λ = 20.86 cm^{-1}; ΔD = 0.30 mm			
Interval of drop diameters, cm	Average diameter D, cm	$N^* = N_0\ exp\,(-\lambda D)$	Number of drops per m^3, $N = N^*\,\Delta D$
0.00 – 0.03	0.015	5851	1755
0.03 – 0.06	0.045	3129	939
0.06 – 0.09	0.075	1674	502
0.09 – 0.12	0.105	895	269
0.12 – 0.15	0.135	479	144
0.15 – 0.18	0.165	256	77
0.18 – 0.21	0.195	137	41
0.21 – 0.24	0.225	73	22
0.24 – 0.27	0.255	39	12
0.27 – 0.30	0.285	21	6
0.30 – 0.33	0.315	11	3
0.33 – 0.36	0.345	6	2
0.36 – 0.39	0.375	3	1
0.39 – 0.42	0.405	2	0
TOTAL:			3773

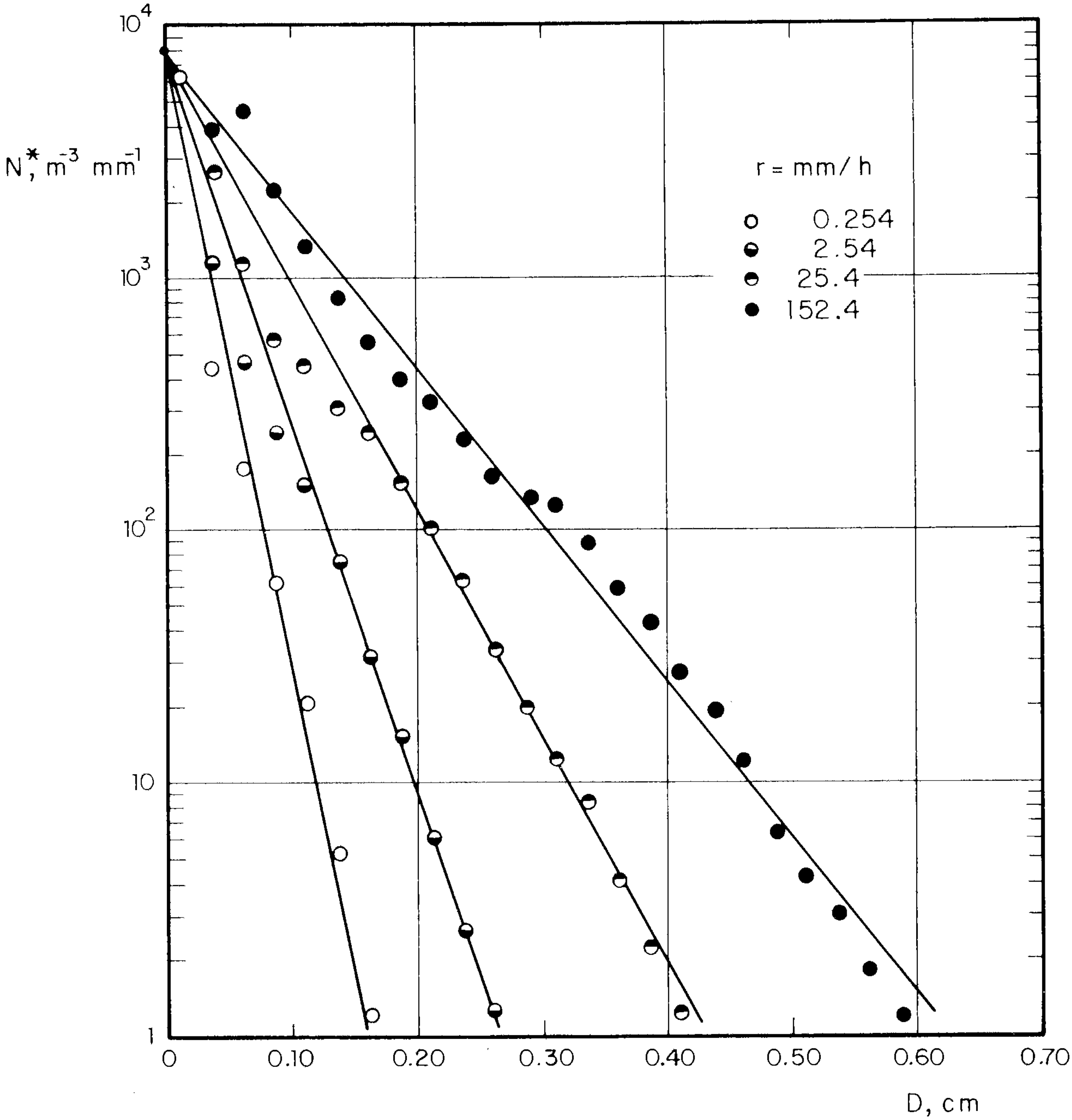

Fig 6.1. Size distribution of raindrops. Data of Laws and Parsons (29)

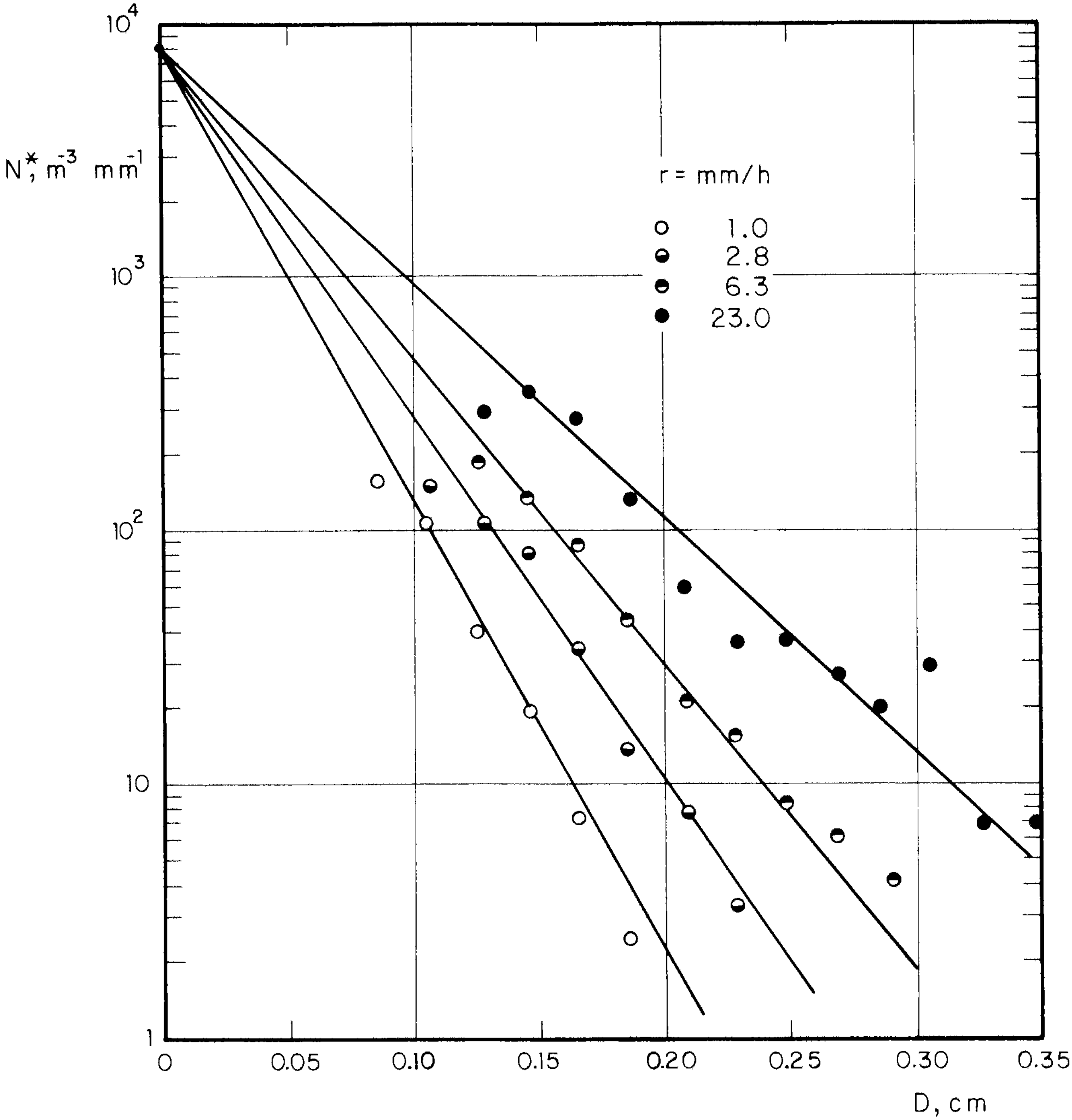

Fig 6.2. Size distribution of raindrops. Data of Marshall and Palmer (32)

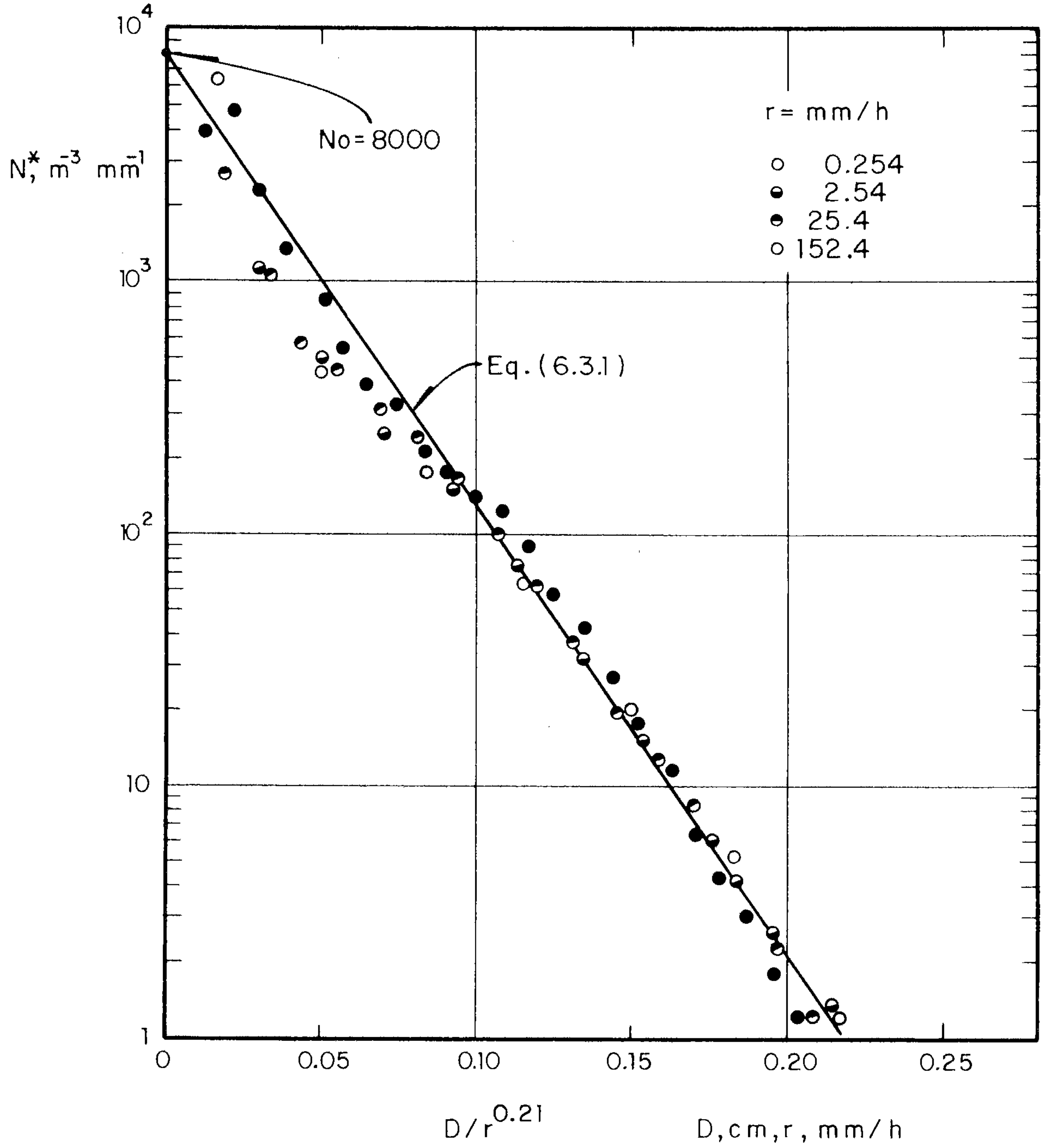

Fig 6.3 *Generalized distribution curve of raindrop diameters. Data of Laws and Parsons (29)*

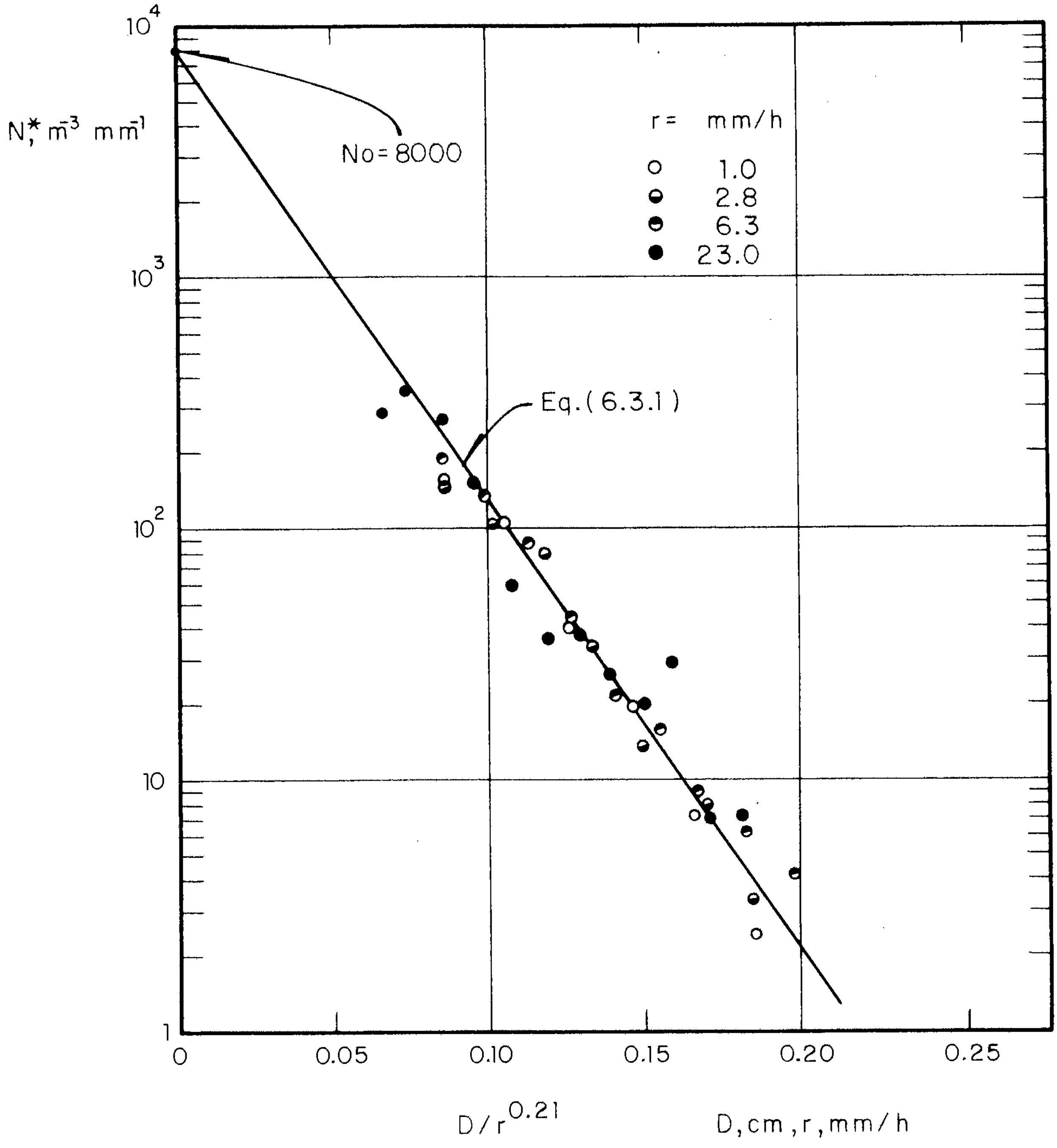

Fig 6.4. *Generalized distribution curve of raindrop diameters. Data of Marshall and Palmer (32)*

7. PARAMETERS OF A RAINFALL

7.1 Introduction

With information available concerning the fall velocities of water drops and the distribution of drop diameters, it is possible to calculate various parameters associated with a rainfall. Among these parameters are (1) the number of drops per unit volume in each drop diameter interval, (2) the number of drops crossing unit horizontal area per unit time, (3) the volumetric concentration of drops, (4) the interval rainfall rates and (5) the energy flux or power.

In the following sections, the results of computations are presented which express the values of these parameters as functions of the drop diameter. Maximum values of the parameters are determined. Finally, the total values of these rainfall parameters are obtained by suming the values corresponding to each drop diameter interval. These total values depend on the rate of rainfall.

7.2 Number of drops per unit volume

The number of drops per unit volume, N (drops/m^3), for a rainfall composed of drops with exponentially-distributed diameters was calculated from eqs. *6.3.1* and *6.3.2*. These computations were made for the following rainfall rates: 1, 5, 10, 25, 50, 100 and 200 mm/h and for the following elevations above sea level: 0, 1,000, 2,000, 3,000, 4,000 and 5,000 m.

The results of the computations for the number of drops per m^3, N, as well as the results corresponding to the other parameters (n, C, r' and P) are presented in Tables 7.1 to 7.6.

D	V	N	n	C	r'	P x 10	N	n	C	r'	P x 10
		r = 1 mm/h					r = 5 mm/h				
0.015	55	1298	707	0.0023	0.00	0.00	1548	844	0.0027	0.01	0.00
0.045	187	379	709	0.0181	0.12	0.06	644	1203	0.0307	0.21	0.10
0.075	304	111	338	0.0245	0.27	0.34	268	815	0.0591	0.65	0.84
0.105	408	32	132	0.0196	0.29	0.67	111	455	0.0675	0.99	2.29
0.135	499	9	47	0.0122	0.22	0.76	46	231	0.0597	1.07	3.71
0.165	578	3	16	0.0065	0.14	0.63	19	111	0.0453	0.94	4.38
0.195	646						8	52	0.0311	0.72	4.29
0.225	703						3	23	0.0199	0.50	3.46
		1832	1949	0.0632	1.04	2.46	2647	3734	0.3163	5.09	18.97
		r = 10 mm/h					r = 25 mm/h				
0.015	55	1643	895	0.0029	0.01	0.00	1755	878	0.0031	0.01	0.00
0.045	187	769	1438	0.0367	0.25	0.12	939	1755	0.0448	0.30	0.15
0.075	304	360	1097	0.0796	0.87	1.12	502	1526	0.1109	1.21	1.56
0.105	408	169	689	0.1023	1.50	3.49	269	1098	0.1628	2.39	5.54
0.135	499	79	395	0.1019	1.83	6.33	144	719	0.1851	3.33	11.52
0.165	578	37	214	0.0871	1.81	8.42	77	445	0.1808	3.77	17.49
0.195	646	17	112	0.0674	1.57	9.07	41	265	0.1596	3.70	21.46
0.225	703	8	57	0.0485	1.23	8.43	22	155	0.1312	3.32	22.79
0.255	752	4	29	0.0330	0.89	7.02	12	90	0.1021	2.82	22.16
0.285	791						6	47	0.0763	2.07	17.99
0.315	823						3	25	0.0551	1.45	13.69
0.345	849						2	17	0.0387	1.31	13.16
0.375	870						1	9	0.0281	0.86	9.09
		3086	4926	0.5594	9.96	44.00	3773	7029	1.2505	26.54	156.60
		r = 50 mm/h					r = 100 mm/h				
0.015	55	1831	998	0.0032	0.01	0.00	1900	1036	0.0034	0.01	0.00
0.045	187	1066	1992	0.0509	0.34	0.17	1190	2224	0.0568	0.38	0.19
0.075	304	621	1890	0.1371	1.50	1.94	746	2270	0.1647	1.81	2.32
0.105	408	361	1476	0.2191	3.22	7.46	467	1907	0.2831	4.16	9.63
0.135	499	210	1050	0.2771	4.87	16.86	293	1461	0.3770	6.77	23.45
0.165	578	123	708	0.2382	6.00	27.84	183	1060	0.4312	8.97	41.65
0.195	646	71	461	0.2769	6.44	37.30	115	742	0.4459	10.37	60.06
0.225	703	42	292	0.2477	6.27	43.10	72	506	0.4291	10.87	74.67
0.255	752	24	182	0.2039	5.68	44.56	45	339	0.3913	10.59	83.06
0.285	791	14	111	0.1706	4.86	42.27	28	223	0.3423	9.75	84.79
0.315	823	8	67	0.1341	3.98	37.45	18	146	0.2895	8.58	80.83
0.345	849	5	41	0.1026	3.14	31.39	11	94	0.2383	7.28	72.89
0.375	870	3	24	0.0767	2.40	25.14	7	60	0.1917	6.00	62.84
0.405	883	2	14	0.0563	1.79	19.40	4	38	0.1513	4.81	52.17
0.435	894						3	24	0.1174	3.78	42.00
0.465	902						2	15	0.0899	2.92	32.99
0.495	908						1	10	0.0679	2.22	25.39
0.525	912						1	6	0.0508	1.69	19.24
		4381	9306	2.2504	50.49	334.85	5086	12161	4.1216	100.93	768.17
		r = 200 mm/h									
0.015	55	1961	1069	0.0035	0.01	0.00					
0.045	187	1309	2445	0.0624	0.42	0.20					
0.075	304	873	2659	0.1929	2.11	2.73					
0.105	408	583	2380	0.3534	5.19	12.02					
0.135	499	389	1943	0.5013	9.01	31.18					
0.165	578	260	1501	0.6109	12.71	59.01					
0.195	646	173	1120	0.6730	15.65	90.66					
0.225	703	116	814	0.6901	17.47	120.08					
0.255	752	77	580	0.6705	18.14	142.31					
0.285	791	52	408	0.6248	17.80	154.78					
0.315	823	34	283	0.5631	16.69	157.19					
0.345	849	23	195	0.4937	15.09	151.04					
0.375	870	15	133	0.4232	13.23	138.70					
0.405	883	10	90	0.3558	11.32	122.70					
0.435	894	7	61	0.2943	9.47	105.25					
0.465	902	5	41	0.2399	7.79	88.06					
0.495	908	3	28	0.1932	6.31	72.21					
0.525	912	2	19	0.1538	5.05	58.30					
0.555	916	1	12	0.1213	4.00	46.55					
0.585	920	1	8	0.0948	3.14	36.90					
0.615	925	1	6	0.0735	2.45	29.14					
0.645	933		4	0.0566	1.90	23.01					
0.675	944		3	0.0433	1.47	18.23					
0.705	959		2	0.0329	1.14	14.54					
0.735	979		1	0.0249	0.88	11.69					
0.765	1005		1	0.0187	0.68	9.52					
0.795	1038		1	0.0140	0.52	7.84					
0.825	1078		1	0.0105	0.41	6.55					
		5895	15807	7.5893	200.08	1710.39					

TABLE 7.1. NUMERICAL VALUES OF THE RAINFALL PARAMETERS

Conditions:

Elevation, $z = 0\ m$

Pressure, $p = 5.7014\ Pa$

Temperature, $T = 15.0$ *Celsius*

Air density, $\rho = 1.226 \times 10^{-3}$

Air viscosity, $\mu = 1.78 \times 10^{-5}$ *dPa-s*

Units:

D: cm — *V: cm/s*

N: drops/m^3 — *n : drops/s-m*^{2}

C: cm^3*/m*^{3}*; r': mm/h* — *P : μJ/s-cm*^{2}

D	V	N	n	C	r'	P x 10	N	n	C	r'	P x 10
		r = 1 mm/h					r = 5 mm/h				
0.015	55	1298	737	0.0023	0.00	0.00	1548	830	0.0027	0.01	0.00
0.045	195	379	739	0.0181	0.13	0.07	644	1254	0.0307	0.22	0.11
0.075	317	111	352	0.0245	0.28	0.39	268	850	0.0591	0.68	0.95
0.105	426	32	138	0.0196	0.30	0.76	111	474	0.0675	1.03	2.60
0.135	520	9	49	0.0122	0.23	0.86	46	241	0.0597	1.12	4.21
0.165	603	3	17	0.0065	0.14	0.71	19	116	0.0453	0.98	4.96
0.195	673						8	54	0.0311	0.75	4.74
0.225	733						3	24	0.0199	0.52	3.92
		1832	2032	0.0832	1.08	2.79	2647	3893	0.3160	5.31	21.49
		r = 10 mm/h					r = 25 mm/h				
0.015	57	1643	933	0.0029	0.01	0.00	1755	998	0.0031	0.01	0.00
0.045	195	769	1499	0.0367	0.26	0.14	939	1829	0.0448	0.31	0.17
0.075	317	360	1144	0.0796	0.91	1.27	502	1594	0.1109	1.27	1.77
0.105	426	169	718	0.1023	1.57	3.94	269	1143	0.1628	2.50	6.28
0.135	520	79	411	0.1019	1.91	7.18	144	748	0.1851	3.47	13.04
0.165	603	37	223	0.0871	1.89	9.53	77	463	0.1808	3.92	19.79
0.195	673	17	117	0.0674	1.63	10.28	41	277	0.1596	3.87	24.36
0.225	733	8	60	0.0485	1.28	9.56	22	161	0.1312	3.46	25.85
0.255	783	4	30	0.0330	0.93	7.94	12	92	0.1021	2.88	24.56
0.285	825						6	52	0.0763	2.27	21.41
0.315	858						3	29	0.0551	1.70	17.42
		3086	5135	0.5594	10.38	49.84	3770	7386	1.2118	25.65	154.65
		r = 50 mm/h					r = 100 mm/h				
0.015	57	1831	1041	0.0032	0.01	0.00	1900	1080	0.0034	0.01	0.00
0.045	195	1066	2077	0.0509	0.36	0.19	1190	2318	0.0568	0.40	0.21
0.075	317	621	1970	0.1371	1.57	2.19	746	2366	0.1647	1.88	2.63
0.105	426	361	1538	0.2191	3.36	8.45	467	1988	0.2831	4.34	10.92
0.135	520	210	1095	0.2711	5.08	19.10	293	1523	0.3770	7.06	26.56
0.165	603	123	738	0.2882	6.25	31.53	183	1105	0.4312	9.35	47.19
0.195	673	71	480	0.2769	6.71	42.26	115	773	0.4459	10.81	68.04
0.225	733	42	305	0.2477	6.54	48.83	72	528	0.4291	11.33	84.58
0.255	783	24	189	0.2099	5.92	50.47	45	353	0.3913	11.04	94.10
0.285	825	14	116	0.1706	5.07	47.89	28	233	0.3423	10.16	96.06
0.315	858	8	70	0.1341	4.15	42.42	18	152	0.2895	8.95	91.57
0.345	885	5	42	0.1026	3.27	35.56	11	98	0.2383	7.59	82.58
0.375	906	3	25	0.0767	2.50	28.49	7	63	0.1917	6.25	71.18
0.405	921						4	40	0.1513	5.02	59.11
0.435	932						3	25	0.1174	3.94	47.11
0.465	940						2	16	0.0889	3.04	37.37
		4379	9686	2.1881	50.77	357.38	5084	12661	4.0019	101.17	819.69
		r = 200 mm/h									
0.015	57	1961	1114	0.0035	0.01	0.00					
0.045	195	1309	2549	0.0624	0.44	0.23					
0.075	317	873	2772	0.1929	2.20	3.09					
0.105	426	583	2481	0.3534	5.41	13.62					
0.135	520	389	2025	0.5013	9.39	35.32					
0.165	603	260	1565	0.6109	13.25	66.86					
0.195	673	173	1167	0.6730	16.31	102.70					
0.225	733	116	848	0.6901	18.22	136.03					
0.255	783	77	605	0.6705	18.91	161.23					
0.285	825	52	425	0.6248	18.55	175.34					
0.315	858	34	295	0.5631	17.40	178.08					
0.345	885	23	203	0.4937	15.73	171.11					
0.375	906	15	139	0.4232	13.80	157.13					
0.405	921	10	94	0.3558	11.80	139.00					
0.435	932	7	64	0.2943	9.88	119.24					
0.465	940	5	43	0.2399	8.12	99.76					
0.495	946	3	29	0.1932	6.58	81.80					
0.525	951	2	19	0.1538	5.26	66.06					
0.555	954	1	13	0.1213	4.17	52.76					
0.585	959	1	9	0.0948	3.27	41.80					
0.615	965	1	6	0.0735	2.55	33.01					
		5895	16465	7.3894	201.27	1834.14					

TABLE 7.2. NUMERICAL VALUES OF THE RAINFALL PARAMETERS

Conditions:

Elevation, $z = 1000\ m$
Pressure, $p = 5.0638\ Pa$
Temperature, $T = 8.5$ *Celsius*
Air density, $\rho = 1.112 \times 10^{-3}\ g/cm^3$
Air viscosity, $\mu = 1.75 \times 10^{-5}\ dPa\text{-}s$

Units:

D: cm — *V: cm/s*
N: drops/m³ — *n: drops/s-m²*
C: cm^3/m^3; r': mm/h; P: $\mu J/s\text{-}cm^2$

D	V	N	n	C	r'	P x 10	N	n	C	r'	P x 10
		r = 1 mm/h					r = 5 mm/h				
0.015	59	1298	768	0.0023	0.00	0.00	1548	916	0.0027	0.01	0.00
0.045	203	379	770	0.0181	0.13	0.08	644	1306	0.0307	0.22	0.00
0.075	331	111	367	0.0245	0.29	0.44	268	885	0.0591	0.70	0.13
0.105	443	32	144	0.0196	0.31	0.85	111	494	0.0675	1.08	1.07
0.135	542	9	51	0.0122	0.24	0.98	46	251	0.0597	1.16	2.94
0.165	628						19	121	0.0453	1.02	4.75
0.195	701						8	56	0.0311	0.79	5.61
		1829	2100	0.0767	0.98	2.35	2644	4029	0.2961	4.99	19.87
		r = 10 mm/h					r = 25 mm/h				
0.015	59	1643	972	0.0029	0.01	0.00	1755	1039	0.0031	0.01	0.00
0.045	203	769	1561	0.0367	0.27	0.15	939	1905	0.0448	0.33	0.19
0.075	331	360	1192	0.0796	0.95	1.44	502	1661	0.1109	1.32	2.00
0.105	443	169	749	0.1023	1.63	4.46	269	1191	0.1628	2.60	7.10
0.135	542	79	429	0.1019	1.99	8.12	144	779	0.1851	3.61	14.75
0.165	628	37	233	0.0871	1.97	10.78	77	483	0.1808	4.09	22.37
0.195	701	17	122	0.0674	1.70	11.62	41	288	0.1596	4.03	27.54
0.225	764	8	62	0.0485	1.33	10.80	22	168	0.1312	3.61	29.24
0.255	816	4	31	0.0330	0.97	8.99	12	96	0.1021	3.00	27.77
0.285	859						6	54	0.0763	2.36	24.20
		3086	5351	0.5594	10.82	56.36	3767	7664	1.1567	24.95	155.16
		r = 50 mm/h					r = 100 mm/h				
0.015	59	1831	1084	0.0032	0.01	0.00	1900	1125	0.0034	0.01	0.00
0.045	203	1066	2163	0.0509	0.37	0.21	1190	2415	0.0568	0.41	0.24
0.075	331	621	2053	0.1371	1.63	2.48	746	2465	0.1647	1.96	2.97
0.105	443	361	1603	0.2191	3.50	9.55	467	2071	0.2831	4.52	12.35
0.135	542	210	1141	0.2711	5.29	21.60	293	1586	0.3770	7.36	30.03
0.165	628	123	769	0.2882	6.51	35.65	183	1151	0.4312	9.75	53.35
0.195	701	71	500	0.2769	6.99	47.78	115	806	0.4459	11.26	76.93
0.225	764	42	317	0.2477	6.81	55.21	72	550	0.4291	11.80	95.64
0.255	816	24	197	0.2099	6.17	57.07	45	368	0.3913	11.50	106.39
0.285	859	14	121	0.1706	5.28	54.14	28	243	0.3423	10.59	108.61
0.315	894	8	73	0.1341	4.32	47.97	18	158	0.2895	9.32	103.53
0.345	922	5	44	0.1026	3.41	40.21	11	102	0.2383	7.91	93.38
0.375	943						7	66	0.1917	6.51	80.48
0.405	959						4	42	0.1513	5.23	66.83
0.435	971						3	26	0.1174	4.11	53.80
		4376	10065	2.1114	50.29	371.87	5082	13174	3.9130	102.23	884.53
		r = 200 mm/h									
0.015	59	1961	1161	0.0035	0.01	0.00					
0.045	203	1309	2655	0.0624	0.46	0.26					
0.075	331	873	2888	0.1929	2.30	3.49					
0.105	443	583	2585	0.3534	5.64	15.40					
0.135	542	389	2110	0.5013	9.78	39.94					
0.165	628	260	1631	0.6109	13.81	75.59					
0.195	701	173	1216	0.6730	16.99	116.13					
0.225	764	116	884	0.6901	18.98	153.80					
0.255	816	77	630	0.6705	19.70	182.29					
0.285	859	52	443	0.6248	19.33	198.25					
0.315	894	34	308	0.5631	18.13	201.35					
0.345	922	23	212	0.4937	16.39	193.47					
0.375	944	15	145	0.4232	14.37	177.66					
0.405	959	10	98	0.3558	12.29	156.98					
0.435	971	7	66	0.2943	10.29	135.00					
0.465	980	5	45	0.2399	8.46	112.79					
0.495	986	3	30	0.1932	6.85	92.50					
0.525	990	2	20	0.1538	5.48	74.68					
		5892	17127	7.0398	199.26	1929.58					

TABLE 7.3. NUMERICAL VALUES OF THE RAINFALL PARAMETERS

Conditions:

Elevation, $z = 2000\ m$
Pressure, $p = 4.4711\ Pa$
Temperature, $T = 2.0$ *Celsius*
Air density, $\rho = 1.007 \times 10^{-3}\ g/cm^3$
Air viscosity, $\mu = 1.72 \times 10^{-5}\ dPa\text{-}s$

Units:

D: cm — *V: cm/s*
N: drops/m^3 — *n: drops/s-m*^{2}
C: cm^3*/m*^{3}*; r': mm/h;* — *P: µJ/s-cm*^{2}

D	V	N	n	C	r'	P x 10	N	n	C	r'	P x 10
		r = 1 mm/h					*r* = 5 mm/h				
0.015	61	1298	792	0.0023	0.01	0.00	1548	945	0.0027	0.01	0.00
0.045	209	379	794	0.0181	0.14	0.08	644	1348	0.0307	0.23	0.14
0.075	341	111	378	0.0245	0.30	0.49	268	913	0.0591	0.73	1.17
0.105	457	32	148	0.0196	0.32	0.94	111	509	0.0675	1.11	3.24
0.135	559	9	53	0.0122	0.25	1.07	46	259	0.0597	1.20	5.21
0.165	648						19	125	0.0453	1.06	6.16
0.195	724						8	58	0.0311	0.81	5.89
		1829	2165	0.0767	1.01	2.58	2644	4157	0.2961	5.14	21.81

D	V	N	n	C	r'	P x 10	N	n	C	r'	P x 10
		r = 10 mm/h					*r* = 25 mm/h				
0.015	61	1643	1003	0.0029	0.01	0.00	1755	1072	0.0031	0.01	0.00
0.045	209	769	1611	0.0367	0.28	0.17	939	1965	0.0448	0.34	0.21
0.075	341	360	1229	0.0797	0.98	1.58	502	1713	0.1109	1.36	2.20
0.105	457	169	772	0.1023	1.68	4.89	269	1229	0.1628	2.68	7.79
0.135	559	79	442	0.1019	2.05	8.91	144	804	0.1851	3.73	16.19
0.165	648	37	240	0.0871	2.03	11.84	77	498	0.1808	4.22	24.56
0.195	724	17	126	0.0674	1.75	12.76	41	298	0.1596	4.16	30.24
0.225	788	8	64	0.0485	1.37	11.85	22	173	0.1312	3.72	32.09
0.255	842						12	99	0.1021	3.10	30.49
0.285	887						6	56	0.0763	2.43	26.57
		3082	5487	0.5265	10.16	52.00	3767	7907	1.1567	25.74	170.34

D	V	N	n	C	r'	P x 10	N	n	C	r'	P x 10
		r = 50 mm/h					*r* = 100 mm/h				
0.015	61	1831	1118	0.0032	0.01	0.00	1900	1160	0.0034	0.01	0.00
0.045	209	1066	2232	0.0509	0.38	0.23	1190	2491	0.0568	0.43	0.26
0.075	341	621	2117	0.1371	1.68	2.72	746	2543	0.1647	2.02	3.27
0.105	457	361	1653	0.2191	3.61	10.49	467	2137	0.2831	4.66	13.55
0.135	559	210	1177	0.2711	5.46	23.71	293	1637	0.3770	7.59	32.97
0.165	648	123	793	0.2882	6.72	39.14	183	1187	0.4312	10.05	58.57
0.195	724	71	516	0.2769	7.21	52.46	115	831	0.4459	11.61	84.45
0.225	788	42	327	0.2477	7.03	60.60	72	567	0.4291	12.17	105.00
0.255	842	24	204	0.2099	6.36	62.66	45	380	0.3913	11.86	116.80
0.285	887	14	125	0.1706	5.45	59.44	28	250	0.3423	10.92	119.23
0.315	923	8	76	0.1341	4.45	52.66	18	163	0.2895	9.62	113.66
0.345	951	5	45	0.1026	3.51	44.13	11	105	0.2383	8.16	102.51
0.375	973						7	68	0.1917	6.72	88.36
0.405	990						4	43	0.1513	5.39	73.36
		4376	10383	2.1114	51.88	408.24	5079	13562	3.7956	101.22	911.99

D	V	N	n	C	r'	P x 10
		r = 200 mm/h				
0.015	61	1961	1197	0.0035	0.01	0.00
0.045	209	1309	2739	0.0624	0.47	0.29
0.075	341	873	2980	0.1929	2.37	3.83
0.105	457	583	2667	0.3534	5.82	16.91
0.135	559	389	2176	0.5013	10.09	43.84
0.165	648	260	1682	0.6109	14.24	82.99
0.195	724	173	1254	0.6730	17.53	127.48
0.225	788	116	912	0.6901	19.58	168.85
0.255	842	77	650	0.6705	20.32	200.12
0.285	887	52	457	0.6248	19.94	217.65
0.315	923	34	317	0.5631	18.70	221.04
0.345	951	23	218	0.4937	16.91	212.40
0.375	973	15	148	0.4232	14.83	195.04
0.405	990	10	101	0.3558	12.68	172.53
0.435	1002	7	68	0.2943	10.61	148.01
0.465	1011	5	46	0.2399	8.73	123.82
0.495	1017	3	31	0.1932	7.07	101.55
		5890	17644	6.9460	199.90	2036.35

TABLE 7.4. NUMERICAL VALUES OF THE RAINFALL PARAMETERS

Conditions:

Elevation, $z = 3000\ m$
Pressure, $p = 3.9460\ Pa$
Temperature, $T = -4.5$ *Celsius*
Air density, $\rho = 0.910\ g/cm^3$
Air viscosity, $\mu = 1.68 \times 10^{-5}\ dPa\text{-}s$

Units:

D: cm — *V: cm/s*
N: drops/m^3 — *n: drops/s-m*^{2}
C: cm^3*/m*^{3}*; r': mm/h;* — *P: μJ/s-cm*^{2}

TABLE 7.5. NUMERICAL VALUES OF THE RAINFALL PARAMETERS

Conditions:

Elevation, $z = 4000\ m$
Pressure, $p = 3.4659\ Pa$
Temperature, $T = -11.0$ *Celsius*
Air density, $\rho = 0.820 \times 10^{-3}\ g/cm^3$
Air viscosity, $\mu = 1.62 \times 10^{-5}\ dPa\text{-}s$

Units:

D: cm — *V: cm/s*
N: drops/m³ — *n: drops/s-m²*
C: cm³/m³; r′: mm/h; — *P: μJ/s-cm²*

r = 1 mm/h

D	V	N	n	C	r'	P x 10
0.015	63	1298	811	0.0023	0.01	0.00
0.045	214	379	813	0.0181	0.14	0.09
0.075	349	111	387	0.0245	0.31	0.52
0.105	468	32	152	0.0196	0.33	1.01
0.135	573	9	54	0.0122	0.25	1.15
0.165	663					
0.195	741					
		1829	2217	0.0767	1.04	2.77

r = 5 mm/h

D	V	N	n	C	r'	P x 10
0.015	63	1548	968	0.0027	0.01	0.00
0.045	214	644	1380	0.0307	0.24	0.15
0.075	349	268	935	0.0591	0.74	1.26
0.105	468	111	522	0.0675	1.14	3.47
0.135	573	46	265	0.0597	1.23	5.60
0.165	663	19	128	0.0453	1.08	6.61
0.195	741	8	59	0.0311	0.83	6.33
		2644	4257	0.2961	5.27	23.42

r = 10 mm/h

D	V	N	n	C	r'	P x 10
0.015	63	1643	1027	0.0029	0.01	0.00
0.045	214	769	1649	0.0367	0.28	0.18
0.075	349	360	1259	0.0796	1.00	1.70
0.105	468	169	791	0.1023	1.73	5.25
0.135	573	79	453	0.1015	2.10	9.57
0.165	663	37	246	0.0871	2.08	12.70
0.195	741	17	129	0.0674	1.80	13.70
0.225	807	8	66	0.0485	1.41	12.75
0.255	862					
0.285	908					
		3082	5620	0.5265	10.40	55.83

r = 25 mm/h

D	V	N	n	C	r'	P x 10
0.015	63	1755	1048	0.0031	0.01	0.00
0.045	214	939	2012	0.0448	0.35	0.22
0.075	349	502	1754	0.1109	1.39	2.36
0.105	468	269	1258	0.1628	2.75	8.37
0.135	573	144	823	0.1851	3.82	17.38
0.165	663	77	510	0.1808	4.32	26.37
0.195	741	41	305	0.1596	4.26	32.46
0.225	807	22	177	0.1312	3.81	34.46
0.255	862	12	101	0.1021	3.17	32.72
0.285	908	6	57	0.0763	2.49	28.53
		3767	8095	1.1567	26.36	182.87

r = 50 mm/h

D	V	N	n	C	r'	P x 10
0.015	63	1831	1145	0.0032	0.01	0.00
0.045	214	1066	2285	0.0509	0.39	0.25
0.075	349	621	2168	0.1371	1.72	2.92
0.105	468	361	1693	0.2191	3.69	11.26
0.135	573	210	1205	0.2711	5.59	25.45
0.165	663	123	812	0.2882	6.88	42.02
0.195	741	71	528	0.2769	7.39	56.32
0.225	807	42	335	0.2477	7.19	65.06
0.255	862	24	208	0.2099	6.52	67.29
0.285	908	14	128	0.1706	5.58	63.82
0.315	945	8	77	0.1341	4.46	56.53
0.345	974	5	46	0.1026	3.60	47.39
0.375	996					
0.405	1014					
		4376	10630	2.1114	53.12	438.28

r = 100 mm/h

D	V	N	n	C	r'	P x 10
0.015	63	1900	1188	0.0034	0.01	0.00
0.045	214	1190	2551	0.0568	0.44	0.28
0.075	349	746	2604	0.1647	2.07	3.51
0.105	468	467	2188	0.2831	4.77	14.54
0.135	573	293	1676	0.3770	7.70	35.40
0.165	663	183	1216	0.4312	10.29	62.88
0.195	741	115	851	0.4459	11.89	90.67
0.225	807	72	581	0.4291	12.46	112.71
0.255	862	45	389	0.3913	12.15	125.40
0.285	908	28	256	0.3423	11.18	128.00
0.315	945	18	167	0.2895	9.85	122.02
0.345	974	11	108	0.2383	8.35	110.05
0.375	996	7	69	0.1917	6.88	94.86
0.405	1014	4	44	0.1513	5.52	78.76
		5079	13888	3.7956	103.64	979.08

r = 200 mm/h

D	V	N	n	C	r'	P x 10
0.015	63	1961	1226	0.0035	0.01	0.00
0.045	214	1309	2805	0.0624	0.48	0.31
0.075	349	873	3051	0.1929	2.43	4.11
0.105	468	583	2731	0.3534	5.96	18.15
0.135	573	389	2228	0.5013	10.33	47.08
0.165	663	260	1722	0.6109	14.58	89.08
0.195	741	173	1284	0.6730	17.95	136.87
0.225	807	116	934	0.6901	20.05	181.26
0.255	862	77	666	0.6705	20.81	214.81
0.285	908	52	468	0.6248	20.42	233.65
0.315	945	34	325	0.5631	19.15	237.31
0.345	974	23	224	0.4937	17.31	228.02
0.375	996	15	153	0.4232	15.18	209.39
0.405	1014	10	104	0.3558	12.98	185.22
0.435	1026	7	70	0.2943	10.87	158.90
0.465	1035	5	47	0.2399	8.94	132.93
0.495	1041	3	32	0.1932	7.24	109.02
		5890	18070	6.9460	204.69	2186.13

TABLE 7.6. NUMERICAL VALUES OF THE RAINFALL PARAMETERS

r = 1 mm/h

D	V	N	n	C	r'	P x 10
0.015	64	1298	826	0.0023	0.01	0.00
0.045	218	379	827	0.0181	0.14	0.09
0.075	355	111	394	0.0245	0.31	0.55
0.105	477	32	154	0.0196	0.34	1.07
0.135	583	9	55	0.0122	0.26	1.21
0.165	675					
0.195	754					
		1829	2256	0.0767	1.05	2.92

r = 5 mm/h

D	V	N	n	C	r'	P x 10
0.015	64	1548	985	0.0027	0.01	0.00
0.045	218	644	1404	0.0307	0.24	0.16
0.075	355	268	952	0.0591	0.76	1.33
0.105	477	111	531	0.0675	1.16	3.65
0.135	583	46	270	0.0597	1.25	5.91
0.165	675	19	130	0.0453	1.10	6.97
0.195	754	8	60	0.0311	0.84	6.67
		2644	4332	0.2961	5.36	24.69

r = 10 mm/h

D	V	N	n	C	r'	P x 10
0.015	64	1643	1045	0.0029	0.01	0.00
0.045	218	769	1678	0.0367	0.29	0.19
0.075	355	360	1281	0.0796	1.02	1.79
0.105	477	169	805	0.1023	1.76	5.54
0.135	583	79	461	0.1019	2.14	10.09
0.165	675	37	250	0.0871	2.12	13.39
0.195	754	17	131	0.0674	1.83	14.44
0.225	821	8	67	0.0486	1.43	13.42
0.255	877					
0.285	924					
		3082	5718	0.5265	10.59	58.86

r = 25 mm/h

D	V	N	n	C	r'	P x 10
0.015	64	1755	1117	0.0031	0.01	0.00
0.045	218	939	2048	0.0448	0.35	0.23
0.075	355	502	1785	0.1109	1.42	2.49
0.105	477	269	1281	0.1628	2.79	8.82
0.135	583	144	838	0.1651	3.68	16.33
0.165	675	77	519	0.1808	4.39	27.80
0.195	754	41	310	0.1596	4.33	34.22
0.225	821	22	181	0.1312	3.88	36.33
0.255	877	12	103	0.1021	3.23	34.50
0.285	924	6	58	0.0763	2.54	30.08
		3767	8240	1.1567	26.83	192.80

r = 50 mm/h

D	V	N	n	C	r'	P x 10
0.015	64	1831	1166	0.0032	0.01	0.00
0.045	218	1066	2326	0.0509	0.40	0.26
0.075	355	621	2207	0.1371	1.75	3.08
0.105	477	361	1703	0.2191	3.76	11.87
0.135	583	210	1226	0.2711	5.69	26.84
0.165	675	123	827	0.2882	7.00	44.30
0.195	754	71	538	0.2769	7.52	59.37
0.225	821	42	341	0.2477	7.32	68.60
0.255	877	24	212	0.2099	6.63	70.91
0.285	924	14	130	0.1706	5.68	67.28
0.315	961	8	79	0.1341	4.64	59.60
0.345	991					
0.375	1014					
0.405	1032					
		4371	10775	2.0088	50.40	412.11

r = 100 mm/h

D	V	N	n	C	r'	P x 10
0.015	64	1900	1209	0.0034	0.01	0.00
0.045	218	1190	2596	0.0568	0.45	0.30
0.075	355	746	2650	0.1647	2.11	3.69
0.105	477	467	2227	0.2831	4.86	15.34
0.135	583	293	1706	0.3770	7.91	37.32
0.165	675	183	1237	0.4312	10.48	66.29
0.195	754	115	866	0.4459	12.10	95.59
0.225	821	72	591	0.4291	12.69	118.84
0.255	877	45	396	0.3913	12.36	132.20
0.285	924	28	261	0.3423	11.38	134.75
0.315	961	18	170	0.2895	10.02	128.85
0.345	991	11	110	0.2383	8.50	116.03
0.375	1014	7	70	0.1917	7.00	100.00
0.405	1032	4	45	0.1513	5.62	83.04
		5079	14134	3.7956	105.49	1032.24

r = 200 mm/h

D	V	N	n	C	r'	P x 10
0.015	64	1961	1248	0.0035	0.01	0.00
0.045	218	1309	2855	0.0624	0.49	0.32
0.075	355	873	3105	0.1929	2.47	4.34
0.105	477	583	2779	0.3534	6.06	19.14
0.135	583	389	2268	0.5013	10.52	49.63
0.165	675	260	1753	0.6109	14.84	93.92
0.195	754	173	1307	0.6730	18.27	144.29
0.225	821	116	950	0.6901	20.40	191.11
0.255	877	77	678	0.6705	21.18	226.52
0.285	924	52	476	0.6248	20.78	246.34
0.315	961	34	331	0.5631	19.49	250.78
0.345	991	23	228	0.4937	17.62	240.41
0.375	1014	15	155	0.4232	15.45	220.75
0.405	1032	10	106	0.3558	13.21	192.29
0.435	1044	7	71	0.2943	11.06	167.52
0.465	1053	5	48	0.2399	9.10	140.15
		5887	18358	6.7528	200.96	2189.91

Conditions:

Elevation, $z = 5000\ m$
Pressure, $p = 3.0383\ Pa$
Temperature, $T = -17.5$ *Celsius*
Air density, $\rho = 0.736 \times 10^{-3}\ g/cm^3$
Air viscosity, $\mu = 1.60 \times 10^{-5}\ dPA\text{-}s$

Units:

D: cm — *V: cm/s*
N: drops/m^3 — *n: drops/s-m*^{2}
C: cm^3*/m*^{3}*; r': mm/h;* — *P:* μ*J/s-cm*^{2}

The distribution of N as a function of drop diameter, D, and rainfall rate, r, is shown in fig 7.1. The drop diameter interval for these and all subsequent computations was $\Delta D = 0.30$ mm.

7.3 Number of drops crossing unit area per unit time

The number of drops crossing a unit horizontal area per unit time, n (drops/s-m^2), was calculated from the following equation

$$n \left(\frac{drops}{s\text{-}m^2}\right) = N \left(\frac{drops}{m^3}\right) V \left(\frac{m}{s}\right) \tag{7.3.1}$$

where V is the fall velocity of a drop. These velocities are given in Table 5.3 and fig. 5.1. Clearly, this parameter, n, represents the number of drops striking each square meter of a body of water or ground surface per second.

The distribution of n as a function of drop diameter and rainfall rate is presented in fig 7.2. The dashed line in the figure identifies the maximum values of n. That such maxima occur is clear from eq. *7.3.1* in which N decreases as the diameter, D, increases whereas V increases as D increases.

7.4 Volumetric concentration of drops

The volumetric concentration of drops, C (cm^3/m^3), was computed from the relationship

$$C \left(\frac{cm^3}{m^3}\right) = N \left(\frac{drops}{m^3}\right) v_o \left(\frac{cm^3}{drop}\right) \tag{7.4.1}$$

where v_o is the volume of a drop of diameter, D. It was assumed that drops are spherical for the entire range of diameters; thus $v_o = \Pi D^3/6$.

The distribution of C is presented in fig 7.3. Again, the dashed line in the figure shows the maximum values of C. For this parameter, it is easy to compute the drop diameter interval containing the maximum amount of water. Substituting eq. *6.3.1* into eq. *7.4.1* gives

$$C = N_o \exp(-\lambda D) \frac{\Pi}{6} D^3 \tag{7.4.2}$$

Differentiating this expression with respect to D and setting the result equal to zero yields

$$D_m = 3/\lambda \tag{7.4.3}$$

where, from eq. *6.3.2*, $\lambda = 41.0/r^{0.21}$ and D_m is the drop diameter corresponding to maximum concentration of water, C_{max}. The value of this maximum concentration is

$$C_{max} = \frac{9\Pi}{2} N_o \frac{exp(-3)}{\lambda^3} \tag{7.4.4}$$

7.5 *Interval rates of rainfall*

The rate of rainfall, r' (cm/s), for each drop diameter interval was determined from the equation

$$r'\left(\frac{cm}{s}\right) = C\left(\frac{cm^3}{m^3}\right) V\left(\frac{m}{s}\right) = n\left(\frac{drops}{s\text{-}m^2}\right) v_o\left(\frac{cm^3}{drop}\right) \tag{7.5.1}$$

The computed values of r', changed to the units of mm/h, are shown in fig 7.4.

7.6 *Power of a rainfall*

The kinetic energy, $E(\mu J)$, of a single drop is

$$E = \frac{10^{-1}}{2} \rho' v_o V^2 \tag{7.6.1}$$

Accordingly, the energy flux or power, P ($\mu J/s\text{-}cm^2$), is

$$P\left(\frac{\mu J}{s\text{-}cm^2}\right) = E\left(\frac{\mu J}{drop}\right) n\left(\frac{drops}{s\text{-}cm^2}\right) = \frac{10^{-1}}{2} \rho' n v_o V^2 = \frac{10^{-1}}{2} \rho' r' V^2 \tag{7.6.2}$$

where, from eq. *7.5.1*, $r' = n v_o$. Computed values of P are presented in fig 7.5. As before, the dashed line shows the maximum values of P and the corresponding drop diameters.

7.7 *Maximum values of the parameters*

The maximum values of the various rainfall parameters (N, n, C, r' and P) and the drop diameters associated with these maximum values are listed in Table 7.7 for rainfall rates ranging from 1 to 200 mm/h. These values correspond to the sea-level condition, $z = 0$.

It is observed that the maximum values invariably occur for different drop diameter intervals. For example, for $r = 25$ mm/h, the maximum number of drops per unit volume, N, corresponds to the limiting case, $D = 0.00$ cm. The maximum number of drops crossing unit area per unit time, n, occurs at the mid-interval diameter, $D = 0.050$ cm. The maximum concentration of water per unit volume, C, corresponds to the diameter, $D = 0.140$ cm. The maximum interval rainfall rate, r', occurs at $D = 0.175$ cm. Finally, the maximum power, P, is generated by drops with the mid-interval diameter, $D = 0.220$ cm.

TABLE 7.7. DIAMETERS OF DROPS, *D* (cm), CORRESPONDING TO INDICATED MAXIMUM VALUES OF RAINFALL PARAMETERS DIAMETER INTERVAL, ΔD = *0.30 mm*

r mm/h	*N* drops/m³		*n* drops/s-m²	*C* cm³/m³	*r'* mm/h	*P* μJ/s-cm²
1	Dia	0.00	0.035	0.075	0.095	0.135
	Value	2400	800	0.025	0.28	0.066
5	Dia	0.00	0.040	0.105	0.135	0.165
	Value	2400	1250	0.068	1.10	0.440
10	Dia	0.00	0.045	0.120	0.150	0.190
	Value	2400	1500	0.103	1.80	0.900
25	Dia	0.00	0.050	0.140	0.175	0.220
	Value	2400	1800	0.185	3.80	2.20
50	Dia	0.00	0.055	0.165	0.195	0.245
	Value	2400	2050	0.295	6.60	4.50
100	Dia	0.00	0.060	0.190	0.225	0.280
	Value	2400	2300	0.450	11.0	8.60
200	Dia	0.00	0.065	0.225	0.255	0.315
	Value	2400	2600	0.700	18.0	16.00

7.8 Total values of the parameters of a rainfall

As mentioned, it was assumed that the distribution of diameters of drops in a rainfall is described by the exponential relationship of eq. *6.3.1.* In order to calculate the total values of the various rainfall parameters it is necessary to compute each drop diameter interval separately and then sum the results. To illustrate the method, a typical computation is shown in Table 7.8. Similar computations were made for rainfall rates, $r = 1, 5, 10, 25, 50, 100$ and 200 mm/h and elevations, $z = 0, 1000, 2000, 3000, 4000$ and 5000 m above sea-leval.

These total values (N_T, n_T, C_T and P_T) were plotted against the rainfall rate, ***r***. The results are shown in fig 7.6 for the sea-level condition, $z = 0$. The lines shown in this figure are of the form

$$N_T,\ n_T,\ C_T,\ P_T = a_i\ r^{m_i} \qquad (7.8.1)$$

The values of a_i and m_i corresponding to each of the four parameters are listed in Table 7.9.

TABLE 7.8. COMPUTATION OF TOTAL VALUES OF RAINFALL PARAMETERS. RAINFALL RATE, $r = 25$ mm/h, ELEVATION, $z = 0$ m
$\lambda = 20.86$ cm^{-1} $\Delta D = 0.30$ mm

D cm	V cm/s	N drops/m^3	n drops/s-m^2	C cm^3/m^3	r' mm/h	P μJ/s-cm^2
0.015	50	1755	878	0.003	0.01	0.000
0.045	187	939	1755	0.045	0.30	0.015
0.075	304	502	1526	0.111	1.21	0.156
0.105	408	269	1098	0.163	2.39	0.554
0.135	499	144	719	0.186	3.33	1.152
0.165	578	77	445	0.181	3.77	1.749
0.195	646	41	265	0.159	3.70	2.146
0.225	703	22	155	0.131	3.32	2.279
0.255	752	12	90	0.104	2.82	2.216
0.285	791	6	47	0.073	2.07	1.799
0.315	823	3	25	0.049	1.45	1.369
0.345	849	2	17	0.043	1.31	1.316
0.375	870	1	9	0.028	0.86	0.909
Total Values		3773	7029	1.276	26.54	15.660

TABLE 7.9. VALUES OF THE COEFFICIENT AND EXPONENTS FOR THE TOTAL VALUES OF RAINFALL PARAMETERS ($z = 0$)

Rainfall parameter	i	a_i	m_i
N_T, drops/m^3	1	1830	0.22
n_T, drops/s-m^2	2	1950	0.40
C_T, cm^3/m^3	3	0.083	0.85
P_T, μJ/s-cm^2	4	0.239	1.26

It is possible to make a comparison of values listed in Table 7.9 for the volumetric concentration, C_T, with results obtained by other investigators. This comparison is summarized in Table 7.10.

TABLE 7.10. VALUES OF THE COEFFICIENT AND EXPONENT FOR THE TOTAL VOLUMETRIC CONCENTRATION, $C_T = a\ r^m$

Investigation	Reference	a	m
Laws and Parsons	(29)	0.068	0.88
Marshall and Palmer	(32)	0.072	0.88
Best	(5)	0.067	0.85
Present study	——	0.083	0.85

Finally, the following empirical equation was obtained to describe the effect of elevation on the total power of a rainfall

$$P_T = (0.239 + 0.103 \times 10^{-4}\ z)\ r^{1.26} \tag{7.8.2}$$

in which z is the elevation in meters above sea-level; the rainfall rate, r, is expressed in mm/h and the total power, P_T, has the units of $\mu J/s\text{-}cm^2$.

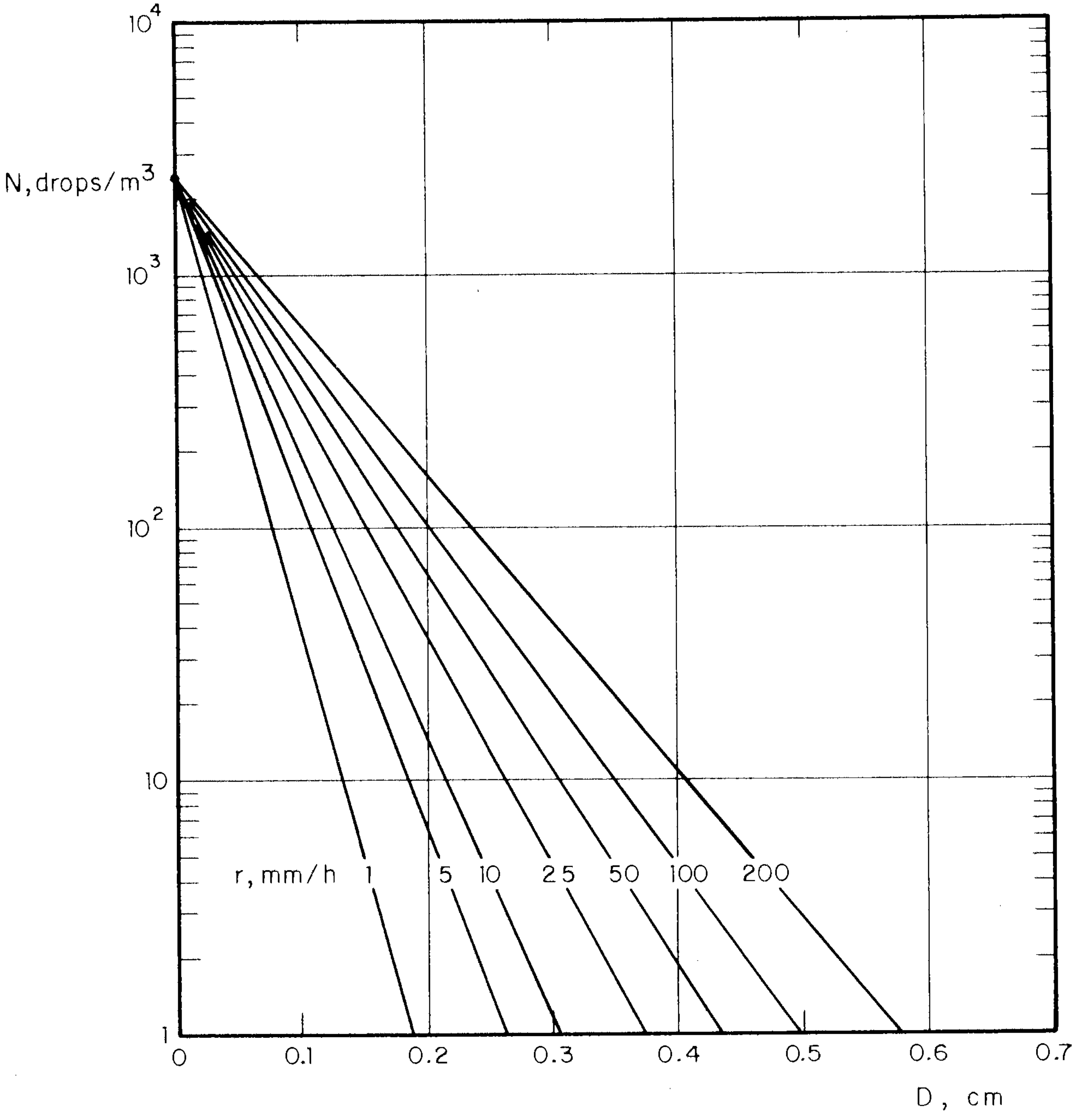

Fig 7.1. Number of drops per unit volume

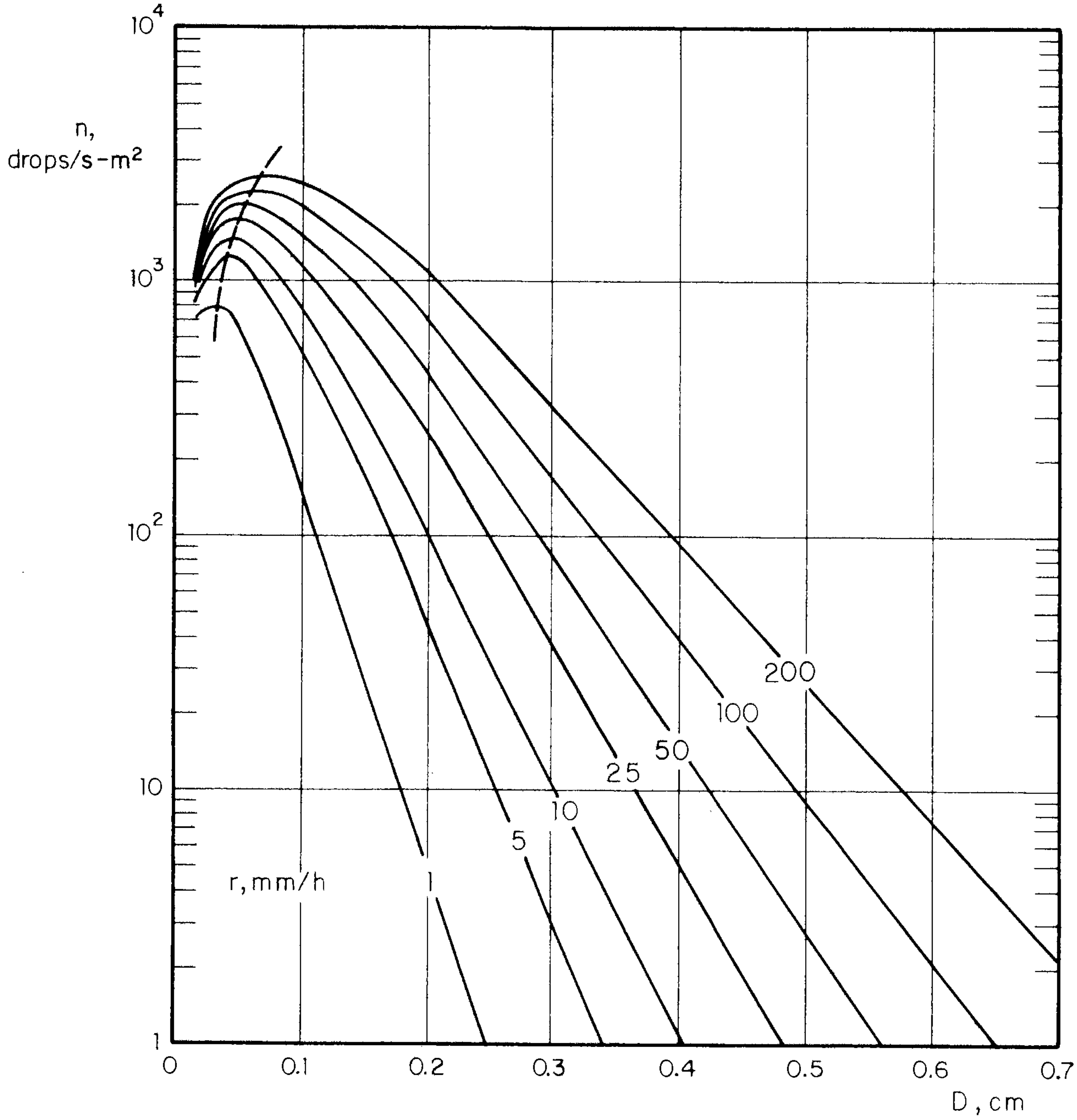

Fig 7.2. Number of drops crossing unit area per unit time

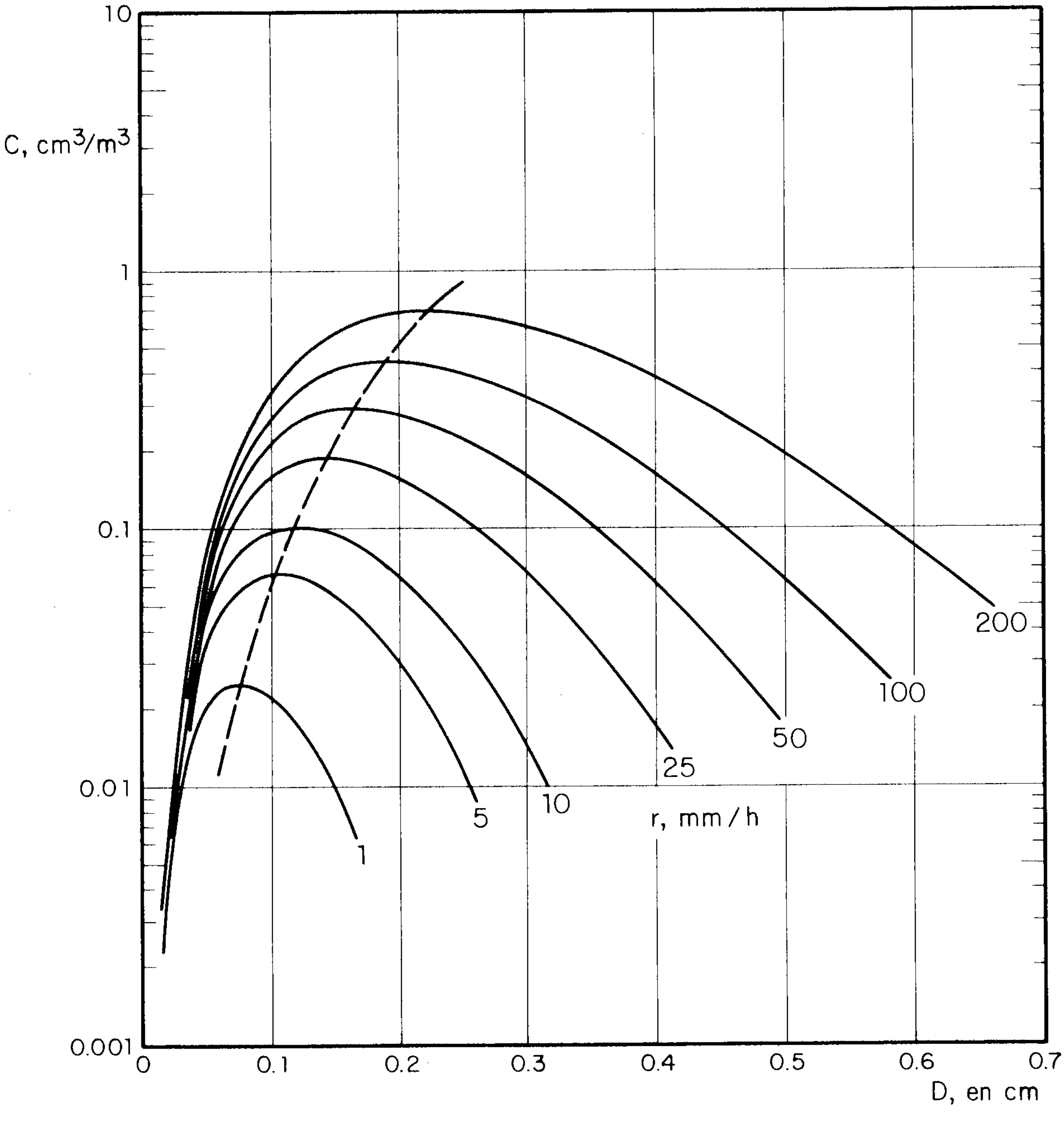

Fig 7.3. Volumetric concentration of drops

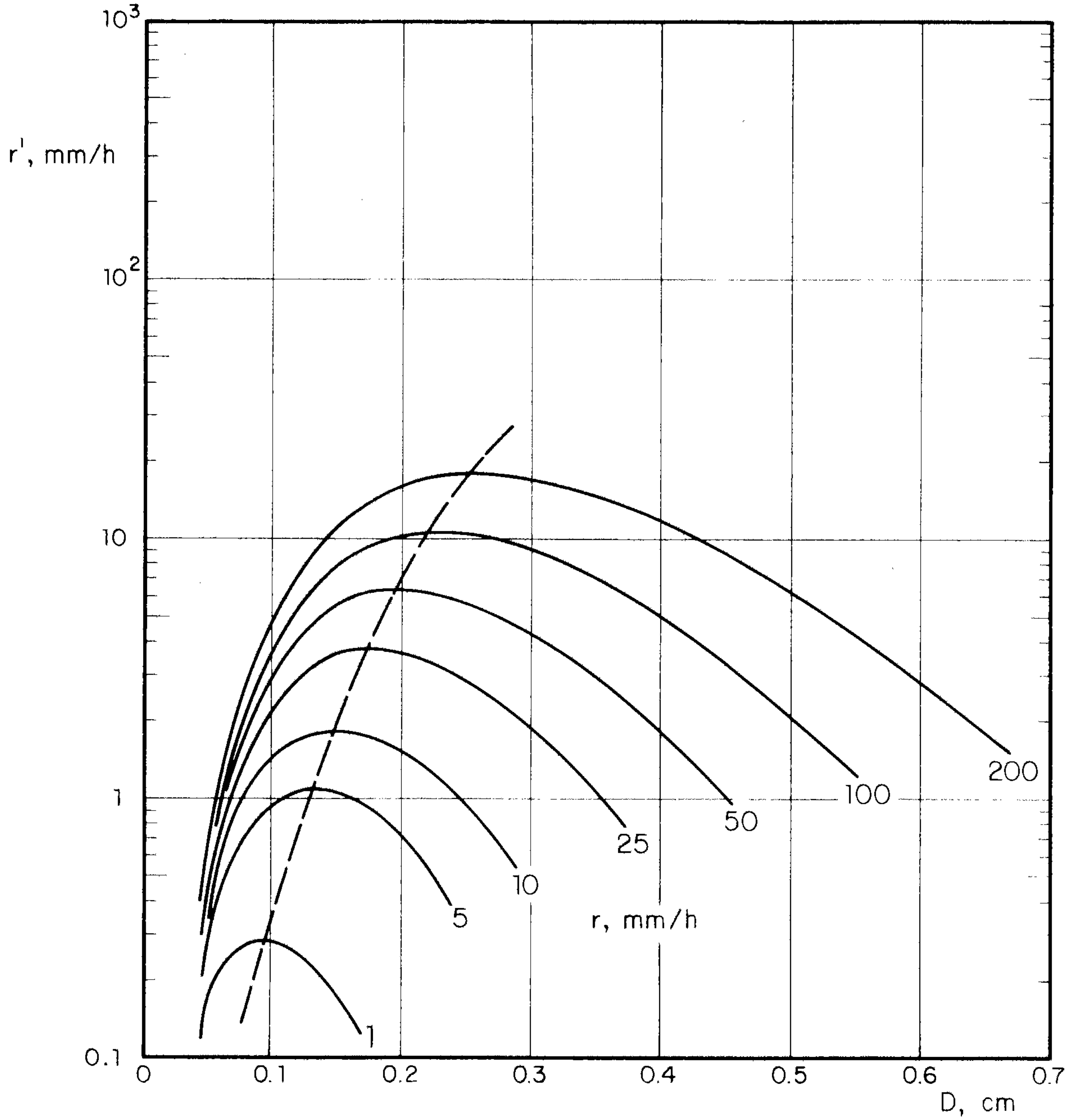

Fig 7.4. Interval rates of rainfall

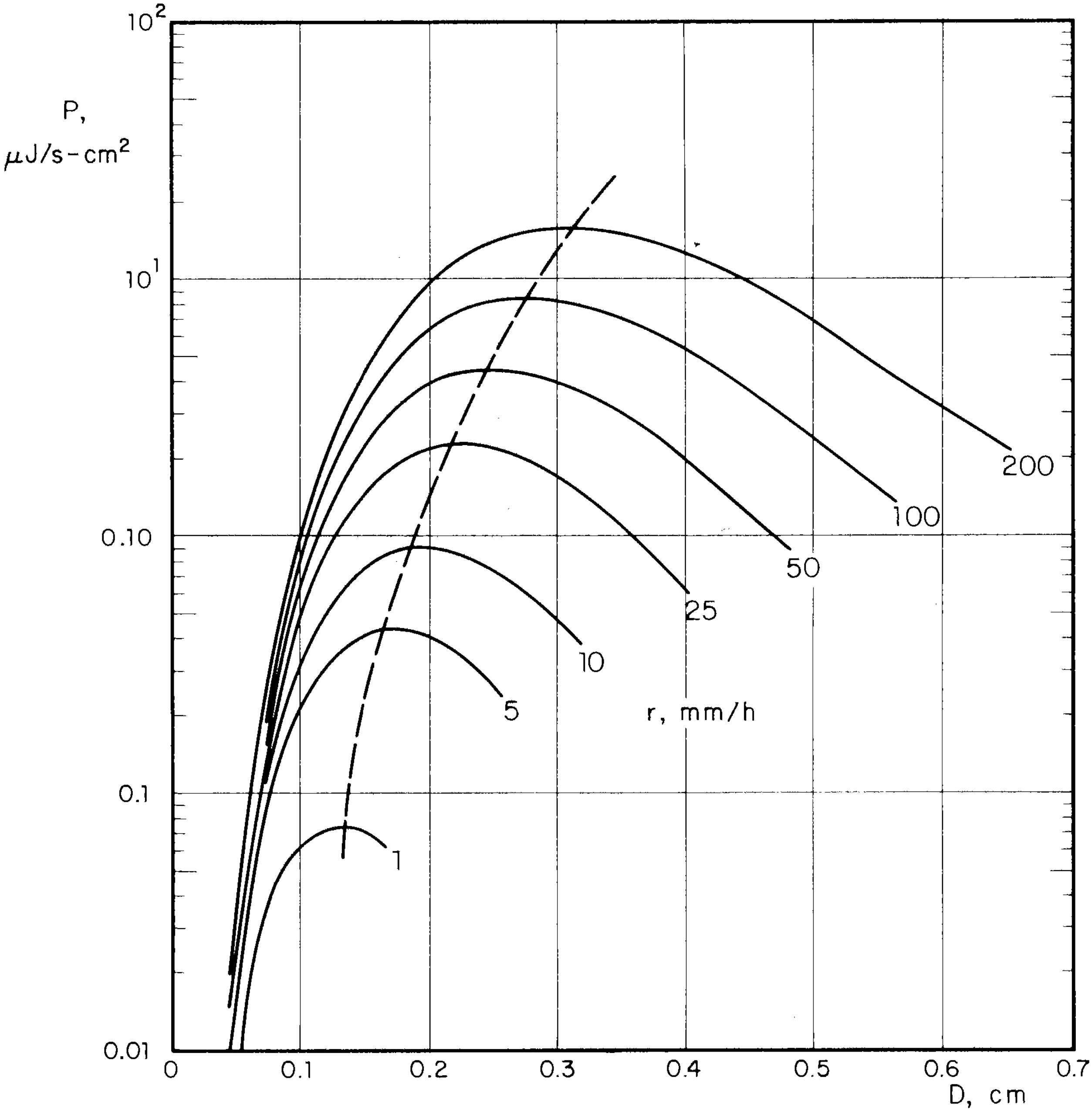

Fig 7.5. Power of a rainfall

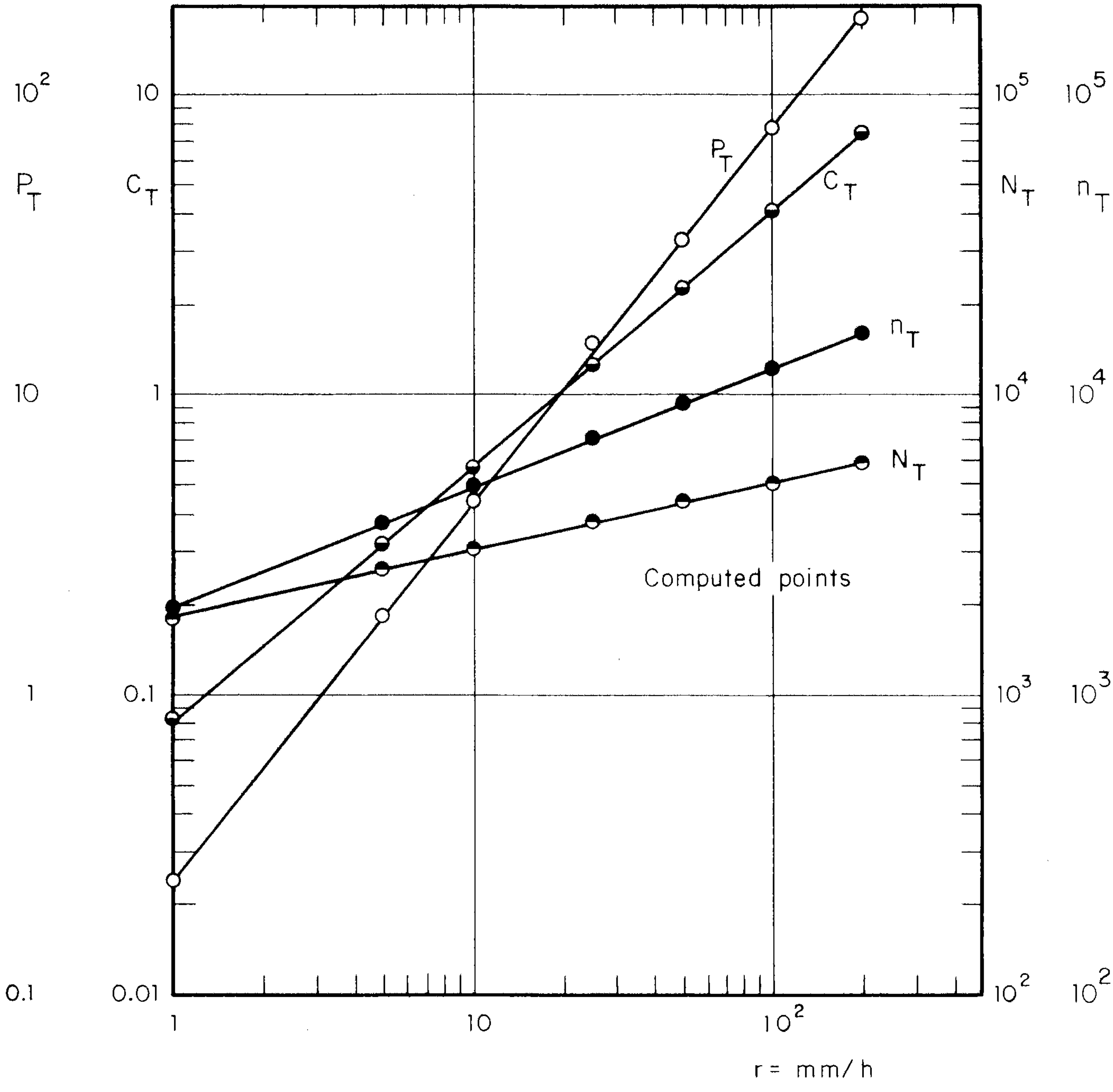

Fig 7.6. Rainfall parameters as functions of the rate of rainfall

8. EFFECT OF RAIN ON SURFACE REAERATION

8.1 Introduction

The following sections are devoted to the task of relating the oxygen transfer coefficient to the power of a rainfall.

Very little research has been conducted on the subject of the effect of rain on surface reaeration. A few experiments were carried out by researchers involved in the study of pollution in the Thames Estuary; the results of that study serve as the basis for the analysis which follows.

The assumption is made that the oxygen transfer coefficient is directly proportional to the rainfall power. Subsequently, a relationship is presented which enables one to determine the total oxygen transfer coefficient, K, if the transfer coefficients due to wind, K_o, and to rainfall K_r, are known.

8.2 Various features of a striking raindrop

The phenomenon of a raindrop striking a water surface has been studied by numerous investigators over a period of many years. The earliest studies along these lines were strictly qualitative in nature. Invariably these early investigations presented simply a description of the features associated with a striking raindrop along with photographs of the phenomenon.

The emphasis of much of this previous work was directed toward the problem of the erosion of soil by rainfall. The publication of Ellison (16) presents a good summary of the earlier studies along these lines. Needless to say, the subject of soil erosion due to rainfall continues to be under very active investigation. Wischmeier and Smith (47) have developed an empirical relationship relating soil loss by erosion to the kinetic energy of a rainfall. Young and Wiersima (50), Yamamoto and Anderson (49) and many other researchers have carried out investigations concerning the soil erosion problem. A publication by Heinemann and Piest (19) presents a survey of past and present research on the phenomenon of splash erosion caused by raindrops.

A number of studies have been carried out during recent years concerning the precise mechanics involved when a raindrop strikes a liquid surface. Typical of such studies are the following:

1. Kilgore and Day (23). These investigators carried out a high-speed photographic study of the striking raindrop. They examined the features of cavity creation, Rayleigh jet emission, secondary droplet formation and capillary wave generation. They determined that the total energy of a drop dissipates exponentially with time after the instant of impact. The order of magnitude of time of energy dissipation was 1.0 s.

2. Chapman and Critchlow (9). This study was concerned with the creation and subsequent motion of a vortex caused by a raindrop striking a liquid surface. Some fraction of the energy of striking drops is utilized to form these vorticies which move in the downward direction and are themselves rapidly dissipated into small scale turbulence and heat.

3. Siscoe and Levin (41). As indicated, some of the energy of the striking drop is consumed in the creation of sub-surface vorticies and associated turbulence. In addition, energy is utilized to create a central cavity, a Rayleigh jet, droplets and surface waves. The research of these investigators was directed toward a study of these various surface phenomena.

4. Bhuiyan, Hiler and Smerdon (6). This research was concerned with the effect of rainfall on the settling velocity of particles in water. It was found that rainfall significantly increases the settling velocities. More importantly, with regard to the present study, these investigators determined that the turbulence intensity caused by striking raindrops is greatly reduced within a layer 7 to 10 cm below the surface.

5. Katsaros (22). This study was concerned with the effect of rainfall on the mixing and dilution of seawater near the surface. The researcher established that these effects are confined to a depth of about *a)* 4 cm for a rainfall corresponding to $D = 1.2$ mm and $r = 4.2$ mm/h and *b)* 10 cm for a rainfall corresponding to $D = 2.7$ mm and $r = 17.0$ mm/h.

6. Mutchler and Larson (34). These investigators carried out an experimental study to determine the amount of droplets created by a raindrop striking a surface of water of various depths. Their study was aimed at the problem of splash erosion. For water depths greater than about 2 cm, their data gave the following result

$$S\left(\frac{g\text{-}droplets}{drop}\right) = k\ E\left(\frac{\mu J}{drop}\right) \tag{8.2.1}$$

where $k = 2.5 \times 10^{-7}$. Multiplying this equation by n (drops/s-cm^2) gives

$$W\left(\frac{g\text{-}droplets}{s\text{-}cm^2}\right) = n\ S = k\ n\ E = k\ P\left(\frac{\mu J}{s\text{-}cm^2}\right) \tag{8.2.2}$$

This result indicates that the wight of droplets produced per unit time and area by a rainfall is directly proportional to the power of the rainfall.

8.3 Effect of rain on the oxygen transfer coefficient

The subject of the effect of rainfall on surface reaeration has received very little attention. Apparently only one such study has been conducted on this topic and even that study was quite limited in scope. That work was carried out as part of the very comprehensive project concerning pollution in the Thames Estuary. The results of the entire project were presented in a report prepared by the Thames Survery Committee and the Water Pollution Research Laboratory (44).

The Thames investigators conducted laboratory experiments to determine the effect of rain on reaeration. In addition, a few tests were made in the field with natural rainfall. In the laboratory experiments, drops of oxygen-saturated water of 5.4 mm diameter were allowed to fall about 3.5 m into a rectangular glass tank containing 40 l of partially de-oxygenated water at a temperature of 20 Celsius. The water in the tank was stirred with an impeller to assure a well-mixed condition. The velocity of the water drops just prior to impact was 7.0 m/s. The drop diameter and velocity were held constant during the experiments. The test variables were the rainfall rate, r, and the initial oxygen transfer coefficient, K_o. The latter quantity is the transfer coefficient due to mechanical mixing only (i.e., the rainfall rate, $r = 0$).

Clearly, the phenomenon of oxygen-saturated drops striking a water surface produces two effects for change of concentration of oxygen in the water. The first effect is the turbulence created by the drops at the air-water interface which result in an increase in the value of the oxygen transfer coefficient. The second effect is the oxygen in the saturated drops which is directly added to the body of water.

In order to determine the true value of the oxygen transfer coefficient, K, it was necessary to resolve these two effects. To accomplish this, the Thames researchers employed the following differential equation

$$\frac{dC}{dt} = \frac{1}{H}\left[K(C_s - C) + C_s r\right] \tag{8.3.1}$$

where H is the depth of water in the tank and r is the rainfall rate. The solution to this equation, with the initial condition $C(0) = C_o$, is

$$\frac{K(C_s - C) + r\,C_s}{K(C_s - C_o) + r\,C_s} = exp\,(-K\,t/H) \qquad (8.3.2)$$

For the rainfall rate, $r = 0$, this equation becomes

$$\frac{C_s - C}{C_s - C_o} = exp\,(-K_o\,t/H) \qquad (8.3.3)$$

The values of K and K_o were determined from eqs. *8.3.2* and *8.3.3* from measurements of the concentration of oxygen at various times after the start of an experiment.

The results of the Thames experiments are presente in Table 8.1. The values shown in the three left columns of the table are the values obtained from a smooth curve through the data given in fig 203 of the Thames report.

TABLE 8.1 OXYGEN TRANSFER COEFFICIENTS OBTAINED IN THE THAMES STUDY

K_o cm/h	r mm/h	K cm/h	$(K - K_o)$ (cm/s) x 10^3	$(K - K_o)/r$
3.0	2.54	5.04	0.564	7.94
	5.08	6.96	1.096	7.77
	7.62	9.09	1.685	7.99
				7.90
5.7	2.54	7.52	0.490	6.90
	5.08	8.72	0.853	6.05
	7.62	9.58	1.074	5.09
				6.01
11.2	2.54	12.54	0.404	5.69
	5.08	13.44	0.715	5.07
	7.62	14.11	0.933	4.42
				5.06
20.9	2.54	22.36	0.383	5.39
	5.08	24.45	0.755	5.36
	7.62	26.33	1.162	5.51
				5.42
38.2	2.54	38.96	0.212	2.99
	5.08	40.49	0.637	4.52
	7.62	42.02	1.061	5.03
				4.18

In fig 8.1, the increase in the value of the oxygen transfer coefficient, $K - K_o$, is plotted against the initial transfer coefficient, K_o, for three rainfall rates, r. In fig 8.2, a correlation is presented between the dimensionless quantity $(K - K_o)/r$, and K_o. Although this correlation is not too precise, it appears that the value of $(K - K_o)/r$ increases as the value of K_o decreases.

The least-squares equation of the line shown in fig 8.2 is

$$\frac{K - K_o}{r} = 6.95 - 282\, K_o \tag{8.3.4}$$

where the units of K, K_o and r are cm/s. Rewritting eq. *8.3.4* gives

$$K = K_o + r\,(6.95 - 282\, K_o) \tag{8.3.5}$$

In obtaining fig 8.2, the 15 data points of fig 8.1 were reduced to 5 points and hence a certain amount of information was lost. In fig 8.3, the 15 points are recovered by a direct comparison of the experimental data and eq. *8.3.5*. The correlation in this figure is fairly good.

8.4 The oxygen transfer coefficient and rainfall power

The Thames experiments were conducted with the drop diameter and terminal velocity held constant ($D = 5.4$ mm and $V = 7.0$ m/s). Accordingly, the energy of each drop was constant with the following value

$$E = \frac{10^{-1}}{2}\, \rho' v_o\, V^2 = \frac{10^{-1}}{2}(1.0)\,\frac{\Pi}{6}(0.54)^3\ (700)^2 = 2\,019\ \mu J \tag{8.4.1}$$

The energy flux or power was varied by changes in the rainfall rate. Thus

$$P = n\, E = \frac{10^{-1}}{2}\, \rho' n\, v_o\, V^2 = \frac{10^{-1}}{2}\, \rho' r\, V^2 \tag{8.4.2}$$

It is observed in eq. *8.3.5* that when $K_o = 0$

$$K = K_r = 6.95\, r \tag{8.4.3}$$

where the subscript, r refers to oxygen transfer due to rainfall. Comparison of eqs. *8.4.2* and *8.4.3* suggests that the transfer coefficient, K_r, is directly proportional to the power P, since both quantities, K_r and P, are linear in r. Therefore, the basic assumption is made that

$$K_r = b_1\, P = b_1 \left(\frac{10^{-1}}{2}\, \rho' r\, V^2 \right) \tag{8.4.4}$$

The value of b_1 was obtained by equating eqs. *8.4.3* and *8.4.4*

$$6.95\, r = b_1 \left(\frac{10^{-1}}{2} \rho' r V^2 \right) \tag{8.4.5}$$

giving $b_1 = 2.83$ cm²/N. It is also assumed that the interaction effect between K_r and K_o, described by the second term in the brackets of eq. *8.3.5*, can be expressed in a similar fashion

$$282\, r = b_2 \left(\frac{10^{-1}}{2} \rho' r V^2 \right) \tag{8.4.6}$$

which gives $b_2 = 115$ s-cm/N. With these assumptions, eq. *8.3.5* may be written in the generalized form

$$K = K_o + \frac{10^{-1}}{2} \rho' r V^2 (b_1 - b_2 K_o) \tag{8.4.7}$$

or from eq. *8.4.2*

$$K = K_o + P(b_1 - b_2 K_o) \tag{8.4.8}$$

Now from eq. *8.4.4*, $K_r = b_1 P$. Accordingly, eq. *8.4.8* can be written in the following form

$$K = (K_o - (K_o K_r / K^*) + K_r) \tag{8.4.9}$$

where $K^* = b_1 / b_2 = 0.0246$ cm/s.

It is recalled that the initial oxygen transfer coefficient, K_o, appearing in the above equations and in fig 8.1, 8.2 and 8.3, corresponds to the oxygen transfer due to mechanical mixing in the experimental tank in the absence of rainfall ($r = 0$). The point-of-view is now shifted from that associated with surface transfer due to mechanical mixing to that identified with surface transfer due to an equivalent wind. Accordingly, from here on, K_o refers to the oxygen transfer coefficient described by eq. *4.3.3*, $K_o = K_o(U)$, where U is the wind velocity.

Consequently, as eq. *8.4.9* shows, the oxygen transfer coefficient, K, depends on the velocity of the wind (U and hence K_o) and on the rate of rainfall (r and hence K_r). The second term on the right hand side of eq. *8.4.9* indicates that there is an interaction between the effects of wind and rain and, accordingly, that K is not simply the sum of K_o and K_r. A graphical presentation of eq. *8.4.9* is given in fig 8.4. It is noted that the curves are symmetrical with respect to the indicated 45° line.

It is possible to express eq. *8.4.9* in the following dimensionless form

$$\left(1 - \frac{K}{K^*}\right) = \left(1 - \frac{K_o}{K^*}\right)\left(1 - \frac{K_r}{K^*}\right) \tag{8.4.10}$$

or simply

$$F = (F_o)(F_r) \tag{8.4.11}$$

where F, F_o and F_r are defined in eq. *8.4.10*. Clearly, eq. *8.4.11* represents a family of hyperbolas with parameter F. A plot of eq. *8.4.11* is presented in fig 8.5.

There are several interesting properties of eq. *8.4.10*. First, if $K_o = 0$ (no wind) then $K = K_r$. Likewise, if $K_r = 0$ (no rain) then $K = K_o$. Second, if $K_o = K^*$ (very strong winds) then $K = K_o = K^* = 0.0246$ cm/s for any value of K_r. From eq. *4.3.3*, this value of K_o corresponds to a wind velocity, $U = 26.4$ m/s $= 95.0$ km/h. By the same token, if $K_r = K^*$ (very strong rains) then $K = K_r = K^* = 0.0246$ cm/s for any value of K_o. From eqs. *8.4.4* and *7.7.2*, this value of K_r corresponds to a rainfall rate, $r = 108$ mm/h. From eq. *8.4.9* one can establish that the maximum value of K is K^* for K_o and K_r greater than zero.

8.5 Summary of the various relationships

A summary of the various relationships obtained thus far is presented in Table 8.2.

TABLE 8.2. SUMMARY OF RELATIONSHIPS FOR DETERMINING VALUES OF THE OXYGEN TRANSFER COEFFICIENTS

Quantity and equation number	Relationship and units
Oxygen transfer coefficient, K_o, due to wind velocity, Eq. *4.3.3*	$K_o = 10^{-6}\left[8.43\sqrt{U} - 3.67\,U + 0.43\,U\right]$ K_o: m/s U: m/s
Oxygen transfer coefficient, K_r, due to rainfall, r Eqs. *8.4.4* and *7.8.2*	$K_r = 2.83\,P$ $P = (0.239 + 0.103 \times 10^{-4}\,z)\,r^{1.26}$ K_r: cm/s P: µJ/s-cm^2 z: m r: mm/h
Total oxygen transfer coefficient, K, due to wind velocity, U, and rainfall, Eq. *8.4.9*	$K = (K_o - (K_o K_r/K^*) + K_r)$ K, K_o and K_r: cm/s $K^* = 0.0246$ cm/s

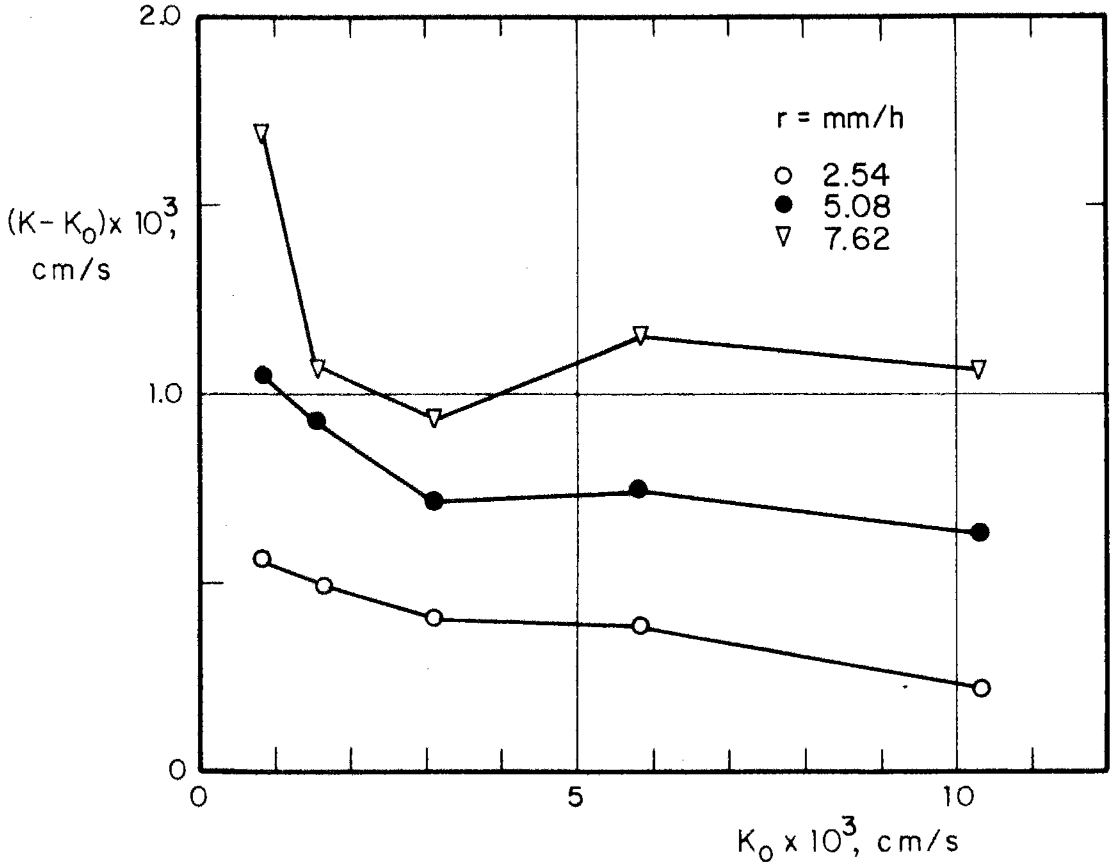

Fig 8.1. *Increase in the value of the oxygen transfer coefficient as a function of the initial transfer coefficient,* K_o, *and the rainfall rate, r*

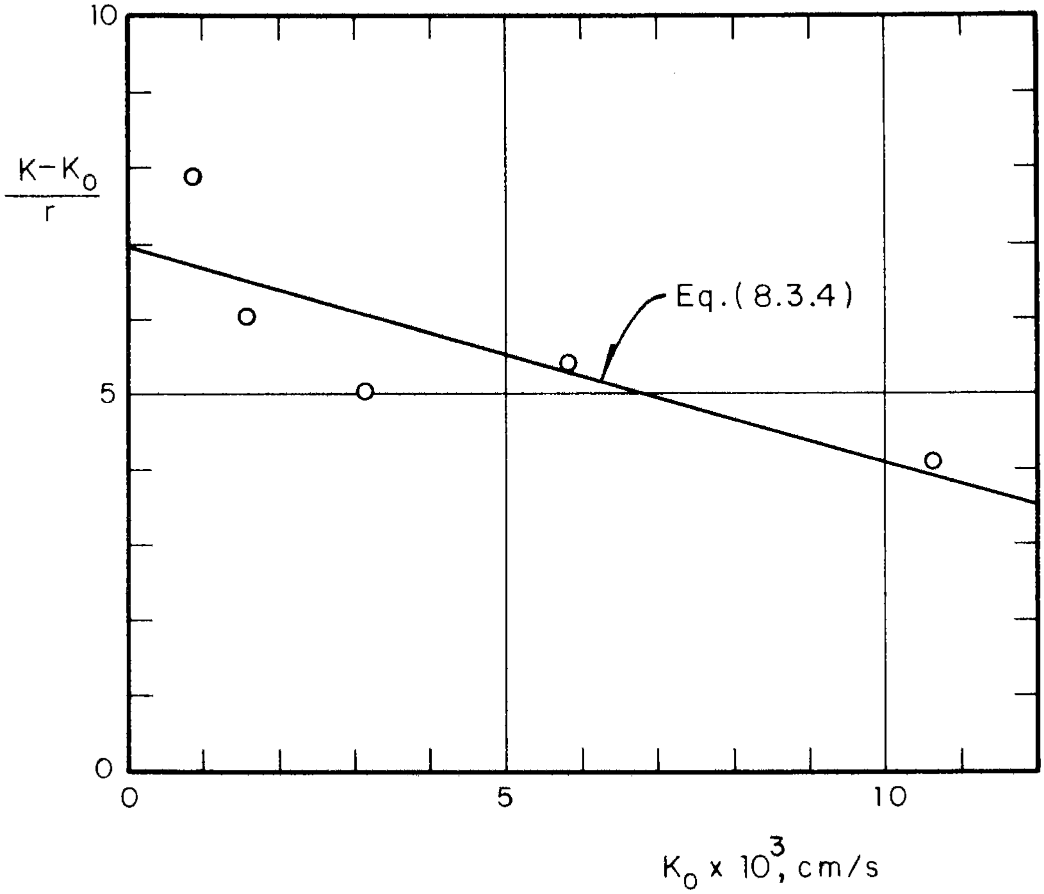

Fig 8.2. *The dimensionless quantity, $(K - K_o)/r$, as a function of the initial transfer coefficient, K_o*

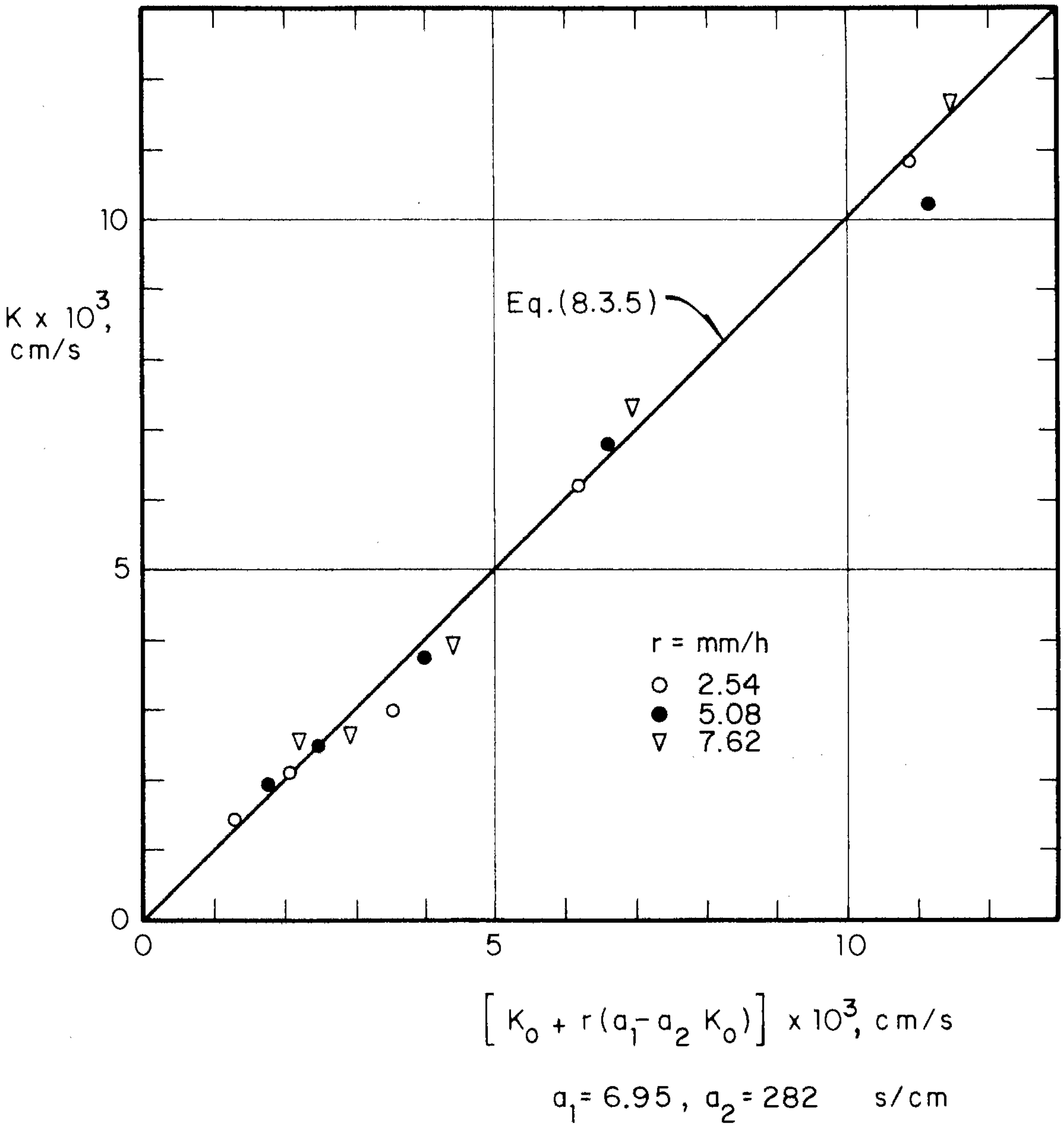

Fig 8.3. Comparison of Eq. (8.3.5) and the experimental data

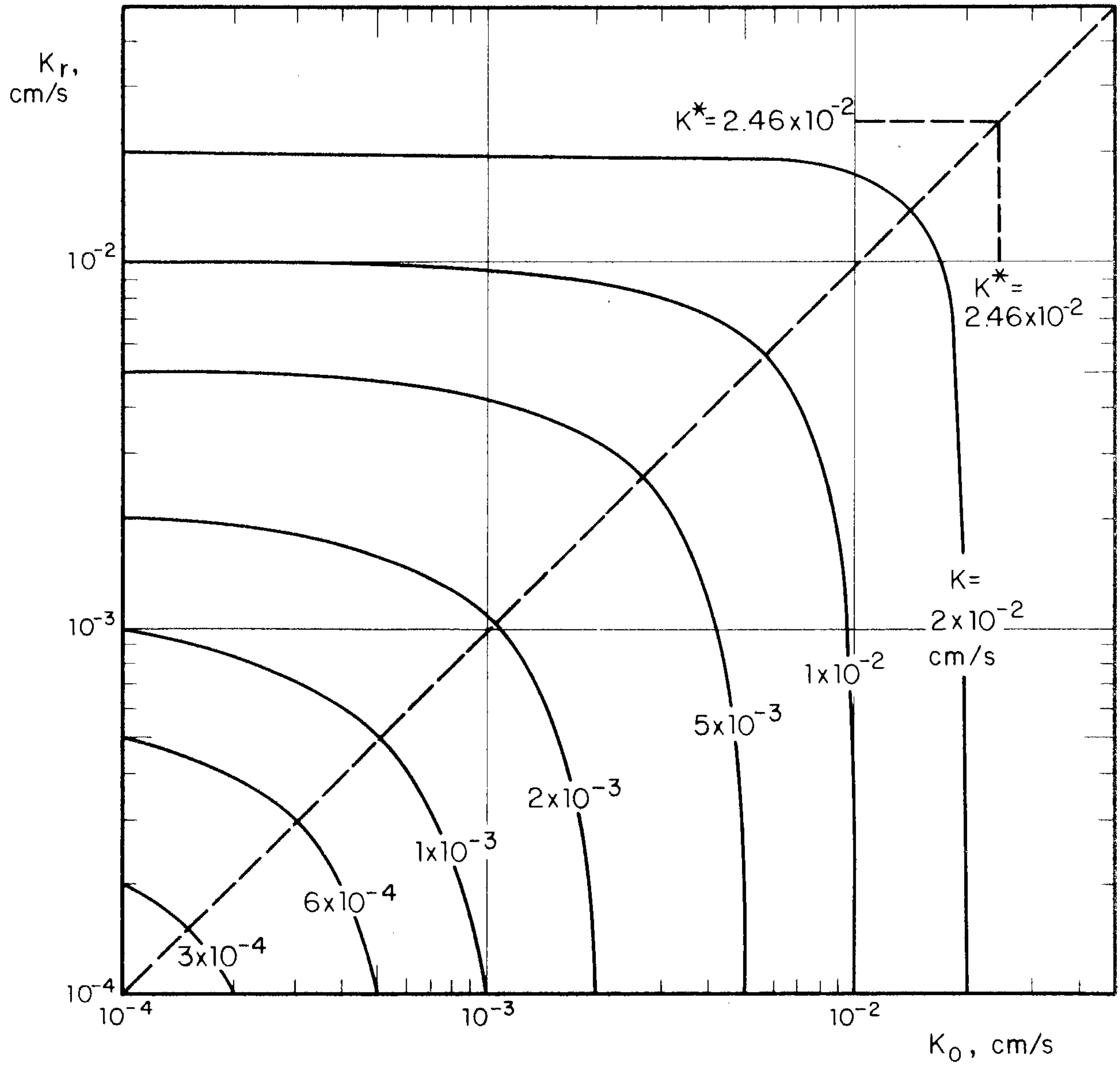

Fig 8.4. *A plot of the function* $K = [K_o - (K_o K_r / K^*) + K_r]$

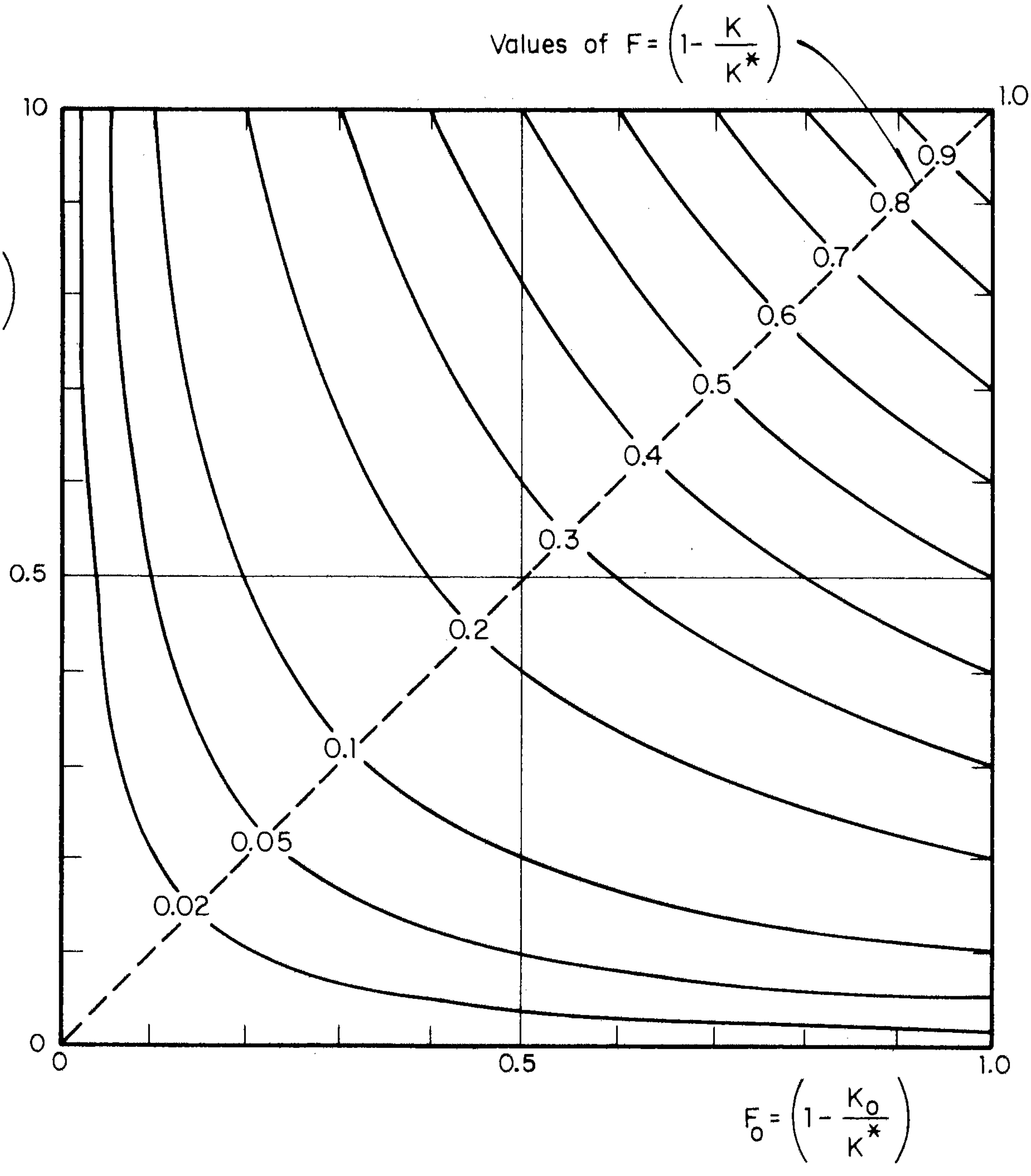

Fig 8.5. *A plot of the function* $\left(1 - \frac{K}{K^*}\right) = \left(1 - \frac{K_o}{K^*}\right)\left(1 - \frac{K_r}{K^*}\right)$

9. EXAMPLES OF APPLICATION

9.1 Introduction

In this chapter, two examples are presented regarding the application of the results given in the previous sections.

The first example involves the effects of wind action and rainfall on the distributions of BOD and DO in a mathematical lake constructed from the complex potential corresponding to an infinite row of sources. In this manner, the velocity distribution was precisely known; the concentrations were computed from the conservation equations in their finite difference forms.

The second example presents information concerning intensities and duration times of rainfalls at Chapala Lake. Values of the surface reaeration coefficients corresponding to maximum rainfalls were calculated. In addition, computations were made of the rates of oxygen transfer to the lake by direct addition of the DO contained in the water drops.

9.2 Example 1: Flow from an infinite row of sources

An example is given below to illustrate the application of the results presented in the previous chapters. This example involves the flow from an infinite row of sources; such a flow pattern corresponds approximately to the flow from a river into a very long lake.

The complex potential, w, for this configuration is given by Milne-Thompson (33).

$$w = -(Q_o/\Pi H) \log_e \sinh(\Pi z/B) \tag{9.2.1}$$

in which B is the distance between the sources located along the y-axis, Q_o is the discharge from a source, H is the depth and $z = x + iy$. Accordingly, the velocity potential and stream function are

$$\phi = -\frac{Q_o}{\Pi H} \log_e (\cosh \lambda x - \cos \lambda y) \tag{9.2.2}$$

and

$$\psi = -\frac{Q_o}{\Pi H} \arctan \left(\frac{\sinh \lambda x \sin \lambda y}{\cosh \lambda x \cos \lambda y - 1} \right) \tag{9.2.3}$$

where $\lambda = 2\,\Pi/B$. Based on these equations, the equipotentials and streamlines are shown in fig 9.1. The velocity components, $u = u(x,y)$ and $v = v(x,y)$ are

$$u = U_o \frac{\sinh \lambda x}{\cosh \lambda x - \cos \lambda y} \tag{9.2.4}$$

and

$$v = U_o \frac{\sin \lambda y}{\cosh \lambda x - \cos \lambda y} \tag{9.2.5}$$

in which $U_o = Q_o/BH$.

The equations for the distribution of BOD and DO, presented in secs. 3.2 and 3.3, are now utilized. Neglecting the effects of dispersion, the steady-state forms of eqs. *3.2.1* and *3.3.1* are

$$u \frac{\partial L}{\partial x} + v \frac{\partial L}{\partial y} + K_1 L = L^* \tag{9.2.6}$$

and

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} + K_1 L + K_2 C = C^* \tag{9.2.7}$$

where, from eqs. *3.2.2* and *3.3.2*

$$L^* = \frac{1}{H} (M_r + M_b) \tag{9.2.8}$$

and

$$C^* = \frac{1}{H}(N_r - N_b) + K_2 C_s + K_p - K_r \tag{9.2.9}$$

To simplify the solution to the above problem, the differential equations are transformed from the rectangular (x,y) coordinate system to a curvilinear (s,n) coordinate system, where the s- and n-directions are parallel to the streamlines and equipotentials, respectively. The resulting expressions, corresponding to eqs. *9.2.6* and *9.2.7*, are

$$U^* \frac{\partial L}{\partial s} + K_1 L = L^* \tag{9.2.10}$$

and

$$U^* \frac{\partial C}{\partial s} + K_1 L + K_2 C = C^* \tag{9.2.11}$$

where $U^* = U^*(s,n)$ is the velocity. Writing these expressions in finite difference form yields

$$L_{i,j} = \frac{L_{i-1,j} + L^* \bar{V}_{i,j}/Q_j}{1 + K_1 \bar{V}_{i,j}/Q_j} \tag{9.2.12}$$

and

$$C_{i,j} = \frac{C_{i-1,j} + (C^* - K_1 L_{i,j}) \bar{V}_{i,j}/Q_j}{1 + K_2 \bar{V}_{i,j}/Q_j} \tag{9.2.13}$$

where i is the number of a particular cell in the s-direction (parallel to the flow), j is the number of a cell in the n-direction (perpendicular to the flow), $\bar{V}_{i,j}$ is the volume of cell (i,j), $L_{i,j}$ and $C_{i,j}$ are, respectively, the concentrations of BOD and DO in cell (i,j) and $Q_j = Q_o/n$ where n is the number of channels in the flow net.

In connection with the problem of flow from an infinite row of sources, the following two cases are considered.

Case A. It is assumed that a wind of variable direction with average velocity, $U = 12.0$ m/s, blows over the surface of the "lake" depicted in fig 9.1. In this case A, there is no rainfall. The following numerical values are selected:

Q_o = $50\ m^3/s;\ n = 12;\ Q_j = Q_o/n = 4.17\ m^3/s$

B == $1\,000\ m;\ H = 10\ m;\ K_1 = 3.47 \times 10^{-6}\ s^{-1} = 0.30\ d^{-1}$

L_o = $10.0\ mg/l;\ C_o = 5.0\ mg/l;\ C_s = 7.0\ mg/l$

M_r = $0;\ M_b = 0;\ \mathbf{L^*} = 0$

N_r = $0;\ N_b = 0;\ K_p = 0;\ K_r = 0$

C^* = $K_{o,2}\ C_s = 3.24 \times 10^{-5}\ mg/l\text{-}s$

The oxygen transfer coefficient due to wind action was determined from eq. *4.3.3;* the result is $K_o = 4.63 \times 10^{-5}$ m/s. With a depth, $H = 10$ m, the reaeration coefficient due to wind action is $K_{o,2} = K_o/H = 4.63 \times 10^{-6}\ s^{-1} = 0.40\ d^{-1}$.

The areas and accordingly the volumes of the cells, $\bar{V}_{i,j}$, were determined from the flow net of fig 9.1.

The numerical values listed above were substituted into eqs. *9.2.12* and *9.2.13* to determine the concentrations, $L_{i,j}$ and $C_{i,j}$. These values were plotted on the flow net and smooth lines of constant concentration were constructed. The results are shown in figs 9.2a and 9.2b for the distributions of BOD and DO, respectively. Details of the above problem are presented in a recent publication by Banks (2).

In fig 9.2a it is seen that the concentration of BOD is reduced from $L_o = 10.0$ mg/l to approximately 1.5 mg/l, along the axis of symmetry $(x = 0)$, in a distance of about 3.0 km. Since $L^* = 0$, the concentration of BOD approaches zero for large values of x.

It is observed in fig 9.2b that the concentration of DO is lowered from $C_o = 5.0$ mg/l to a minimum value of approximately 3.2 mg/l in a distance of about 1.0 km. Since $C^* = K_2\, C_s$, the asymptotic concentration of DO is equal to the equilibrium concentration $C_s = 7.0$ mg/l.

Case B. It is now assumed that a rainfall of rate, $r = 25$ mm/h, occurs over the lake. In addition, as in case A, a wind of velocity, $U = 1.20$ m/s, blows over the lake.

The following numerical values are selected; most of these values are the same as in case A:

$$Q_o = 50\ m^3/s; \quad n = 12; \quad Q_j = Q_o/n = 4.17\ m^3/s$$

$$B = 1000\ m; \ H = 10\ m; \ K_1 = 3.47 \times 10^{-6}\ s^{-1} = 0.30\ d^{-1}$$

$$L_o = 10.0\ mg/l; \ C_o = 5.0\ mg/l; \ C_s = 7.0\ mg/l$$

$$M_r = 0; \quad M_b = 0; \quad L = 0$$

$$N_r = C_s\, r = 0.486 \times 10^{-5}\ mg/s\text{-}cm^2; \quad N_b = 0; \quad K_p = 0; \quad K_r = 0$$

The power of the rainfall was computed from eq. *7.8.2*, with the elevation, $z = 0$; the result is $P = 13.8\ \mu J/s\text{-}cm^2$. Subsequently, the oxygen transfer coefficient due to rainfall was determined from eq. *8.4.4* with the result $K_r = 3.91 \times 10^{-5}$ m/s. With a depth, $H = 10$ m, the reaeration coefficient due to rainfall is $K_{r,2} = K_r/H = 3.91 \times 10^{-6}\ s^{-1} = 0.34\ d^{-1}$.

The total oxygen transfer coefficient, K, due to the combined effects of wind action and rainfall was calculated from eq. *8.4.9*. The resulting value is $K = 7.80 \times 10^{-5}$ m/s. With $H = 10$ m, the total reaeration coefficient is $K_2 = 7.80 \times 10^{-6}\ s^{-1} = 0.67\ d^{-1}$.

Finally, the value of C^* was determined from eq. *9.2.9*.

$$C^* = \frac{1}{D}(N_r - N_b) + K_2\, C_s + K_p - K_r = (0.49 \times 10^{-5}) + (5.46 \times 10^{-5}) = 5.95 \times 10^{-5}\ mg/l\text{-}s$$

In the preceeding equation, the quantity in the first set of parentheses represents the direct addition of DO contained in the oxygen-saturated drops of water. The quantity in the second set of parentheses reflects the effect of wind and rain on the surface reaeration coefficient.

As in case A, the above-indicated numerical values were substituted into eqs. *9.2.12* and *9.2.13* to compute $L_{i,j}$ and $C_{i,j}$. The results for $L_{i,j}$, shown in fig 9.2a, are the same as in case A. The results for $C_{i,j}$ are presented in fig 9.2c.

A comparison of figs 9.2b and 9.2c shows the effect of rainfall. It is observed that the minimum concentration, $C = 4.4$ mg/l, is substantially larger in case B. Furthermore, the location of this minimum value is somewhat closer to the source in case B than in case A. At a distance of 3.0 km along the axis of symmetry, the concentration of DO is about 6.5 mg/l in case B, compared to approximately 4.7 mg/l in case A.

For large values of x, the DO concentration, in case B, is $C = C^*/K_2 = 7.63$ mg/l. Accordingly, in this region, the water is super-saturated ($C_s = 7.0$ mg/l) and hence there is a transfer of oxygen from the lake to the atmosphere. The rate of this transfer is equal to the rate of direct addition to the lake of DO contained in the oxygen-saturated raindrops. In the preceeding computation the increase in the rate of flow, Q (m^3/s), due to the rainfall was neglected.

The following Table 9.1 presents a summary of the values of the various coefficients employed in the example.

TABLE 9.1. SUMMARY OF THE VALUES OF THE SELECTED OR CALCULATED COEFFICIENTS IN CASES A AND B

Coefficient	Symbol	Units	Case A	Case B
Deoxigenation coefficient	K_1	s^{-1}	3.47×10^{-6}	3.47×10^{-6}
	K_1	d^{-1}	0.30	0.30
Oxygen transfer coefficient	K_o	m/s	4.63×10^{-5}	4.63×10^{-5}
Reaeration coefficient due to wind	$K_{o,2}$	s^{-1}	4.63×10^{-6}	4.63×10^{-6}
		d^{-1}	0.40	0.40
Oxygen transfer coefficient due to rain	K_r	m/s	0.00	3.91×10^{-5}
Reaeration coefficient due to rain	$K_{r,2}$	s^{-1}	0.00	3.91×10^{-6}
		d^{-1}	0.00	0.34
Total oxygen transfer coefficient due to wind and rain	K	m/s	4.63×10^{-5}	7.80×10^{-5}
Total reaeration coefficient due to wind and rain	K_2	s^{-1}	4.63×10^{-6}	7.80×10^{-6}
		d^{-1}	0.40	0.67

9.3 Example 2: Chapala Lake

A second example is presented below regarding some effects of rainfall on surface reaeration in Chapala Lake.

A map of the region of Chapala Lake is shown in fig 9.3. This lake, the largest in Mexico, is located about 50 km south of Guadalajara City. Its length (east-to-west) is approximately 80 km and its width (north-to-south) ranges from 6 to 25 km. Its average depth is about 8 m and does not vary greatly from place to place. The surface area of the lake is approximately 1 140 km^2 and its volume is around 6.8 x 10^9 m^3.

Only two rivers of significance are involved in the hydrology of the lake: the Lerma River that enters at the eastern end and the Santiago River that flows from the lake at its north-eastern corner, less than 15 km from the mouth of the Lerma. The average discharges of the Lerma and Santiago over the 20-year period, 1953-1972, were 50.0 m^3/s and 37.2 m^3/s, respectively. Over the same period of time, the average rate of evaporation was 53.0 m^3/s. Nearly all of the rainfall takes place during the months of June through October. The highest rates of evaporation occur during the months of March through June.

Information concerning monthly average rates of rainfall and evaporation on Chapala Lake is presented in Table 9.2. This information was obtained from reports prepared by the Institute of Engineering (21) of the National Autonomous University of Mexico.

TABLE 9.2. MONTLY AVERAGE RATES OF RAINFALL AND EVAPORATION, CHAPALA LAKE, 1953-1972

M o n t h	Rainfall, mm/mon	Evaporation, mm/mon
January	11.2	113
February	5.3	144
March	4.0	198
April	7.9	222
May	24.8	238
June	163.8	192
July	194.6	162
August	156.7	154
September	137.4	141
October	55.6	132
November	9.6	114
December	7.9	99
Annual	778.8	1 909

During the months from December to May, the prevailing winds over the lake are from east-to-west at an average velocity of about 10 km/h. From June to November, the dominant winds are from west to-east with an average velocity of approximately 9 km/h. The elevation of the lake is 1 525 m above sea-level.

For numerous years, the Secretaría de Recursos Hidráulicos (SRH)* and the Secretaría de Obras Públicas (SOP)** have collected data concerning rainfall at many localities throughout Mexico. A report prepared by the SOP (40) presents considerable information concerning rainfall at 11 stations in the region of Chapala Lake. The maximum observed precipitation in 24 hours at these 11 stations is shown in Table 9.3. It is noted that the average value of the maximum 24-hour rainfall at these stations is about 105 mm.

TABLE 9.3. MAXIMUM RAINFALL DURING 24 HOURS AT 11 STATIONS NEAR CHAPALA LAKE, MEXICO

Station	Years of record	Years max observed	Maximum r, mm/24 h	Maximum r, mm/h
La Raya, Mich.	29	1969	96.6	4.03
La Palma, Mich.	29	1967	82.0	3.42
Comoato, Mich.	23	1970	93.9	3.91
Briseñas, Mich.	22	1941	110.0	4.58
Tuxcuenca, Jal.	32	1962	110.0	4.58
Chapala, Jal.	40	1967	121.7	5.07
Jamay, Jal.	31	1972	80.0	3.33
La Barca, Jal.	19	1944	70.0	2.92
El Fuerte, Jal.	28	1966	119.0	4.96
Poncitlán, Jal.	38	1973	109.0	4.54
Atoyac, Jal.	31	1946	166.0	6.92
Average			105.3	4.39

The report of the SOP also presents information regarding observed maximum intensities of rainfall at Chapala City for duration times, T, ranging from 5 minutes to 24 hours. This information is summarized in Table 9.4 and is presented in graphical form in fig 9.4.

TABLE 9.4. MAXIMUM RAINFALLS AT CHAPALA CITY

T, min	T, h	R, mm/T	r, mm/h
5	0.083	18.0	216.0
10	0.167	25.5	135.0
20	0.333	38.0	114.0
30	0.50	48.8	97.6
60	1.00	62.5	62.5
120	2.00	87.8	43.9
360	6.00	95.0	15.8
720	12.00	97.6	8.1
1 440	24.00	108.0	4.5

* Actually, Secretaría de Agricultura y Recursos Hidráulicos (SARH)

** Actually, Secretaría de Asentamientos Humanos y Obras Públicas (SAHOP)

The equation of the line shown in fig 9.4 is

$$r = 50.5/T^{0.72} \tag{9.3.1}$$

where the units of the rainfall rate, r, and duration time, T, are mm/h and h, respectively. For comparison, the correlation between values of world-wide maximum rainfall rates and corresponding duration times is shown as the dashed line in fig 9.4. The equation of this line, developed from data presented by Eagleson (14), is

$$r = 389/T^{0.514} \tag{9.3.2}$$

It is observed from fig 9.4 that maximum rainfall rates at Chapala Lake are about an order-of-magnitude less than world-wide values.

The report of the SOP presented the results of a probability analysis of maximum rainfall rates as a function of the return period, T_r. The information contained in the SOP report, corresponding to the region of Chapala Lake, produced the following relationship

$$r = (25.64\ T_r^{0.19})/T^{0.72} \tag{9.3.3}$$

where the return period, T_r, is expressed in years. Comparison of eqs. *9.3.1* and *9.3.3* indicated that the return period implicit in eq. *9.3.1* is about 35 years.

To illustrate an application of the above information, the following numerical values were selected.

Return period, T_r: *25 yrs* — Elevation, z: *1 525 m*

Depth, H: *8 m* — Temperature, T: *20 Celsius*

Equilibrium concentration of oxygen, C_s: *7.7 mg/l*

Accordingly, from

eq. *9.3.3* $r = 47.26/T^{0.72}$
eq. *7.8.2* $P = 0.255\ r^{1.26}$
eq. *8.4.4* $K_r = 2.83\ P$
eq. *4.2.3* $K_{r,2} = K_r/H$

The results of computations are given in Table 9.5. It is noted that eq. *9.3.3* was extrapolated to larger values of duration time, T.

Numerical values of the dimensionaless quantity, $G_r = T\,K_{r,2}$, are shown in Table 9.5. From the equations listed above it can be shown that

$$G_r = T\ K_{r,2} = 0.0418\ T^{0.093} \tag{9.3.4}$$

TABLE 9.5. VALUES OF RAINFALL PARAMETERS, CHAPALA LAKE

T h	r mm/h	P $\mu J/s\text{-}cm^2$	K_r $(m/s \times 10^5)$	$K_{r,2}$ d^{-1}	G_r $T(h) K_{r,2}$ (h^{-1})
0.083	283	312.8	88.52	9.56	0.0330
0.167	171	165.8	46.92	5.07	0.0352
0.333	104	88.6	25.07	2.71	0.0376
0.50	77.8	61.5	17.40	1.88	0.0392
1.00	47.3	32.8	9.28	1.00	0.0418
2.00	28.7	17.5	4.95	0.535	0.0446
6.00	13.0	6.45	1.83	0.197	0.0493
12.00	7.9	3.44	0.97	0.105	0.0526
24.00	4.8	1.84	0.52	0.056	0.0562
48.00	2.91	0.978	0.277	0.0311	0.0622
120.00	1.505	0.426	0.121	0.0130	0.0651
240.00	0.914	0.227	0.064	0.00695	0.0695
480.00	0.555	0.121	0.034	0.00370	0.0741
1200.00	0.287	0.053	0.015	0.00161	0.0805

The quantity, G_r, reflects the combined effects of time of duration of a rainfall and the intensity of the rainfall. Thus, the amount of oxygen transferred across a unit area of the surface in time T, due to interfacial turbulence caused by the drops, is

$$W = M T = K_{r,2} H (C_s - C) T = H (C_s - C) G_r \qquad (9.3.5)$$

As indicated in eq. *9.3.4*, G_r, although not constant, is a rather insensitive increasing function of T. In essence this fact, along with eq. *9.3.5*, indicates that the total amount of oxygen transferred by interfacial turbulence in a long duration-low intensity rainfall is not greatly more than that transferred during a short-duration-high intensity rainfall. It should be emphasized that the above results refer to the rainfalls described by eq. *9.3.3*.

As mentioned before, a rainfall contributes oxygen to a body of water in two ways: *a)* by creating turbulence at the air-water interface resulting in an increase in the oxygen transfer coefficient and *b)* by the direct addition of oxygen contained in the water drops. The amount contributed by direct addition, W, in duration time, T, can be computed from the following equation

$$W = M T = (C_s)(r)(T) \qquad (9.3.6)$$

in which it is assumed that the drops are oxygen-saturated. The results of computations of oxygen directly added to Chapala Lake during maximum-type rainfalls are shown in Table 9.6.

TABLE 9.6. DIRECT ADDITION OF OXYGEN BY RAINDROPS, CHAPALA LAKE

T h	r mm/h	W g/m^2	ΔC mg/l
0.083	283	0.182	0.023
0.167	171	0.220	0.028
0.333	104	0.267	0.033
0.50	77.4	0.300	0.038
1.00	47.3	0.364	0.046
2.00	28.7	0.442	0.053
6.00	13.0	0.601	0.075
12.00	7.9	0.730	0.091
24.00	4.8	0.887	0.111
48.00	2.91	1.076	0.134
120.00	1.505	1.391	0.174
240.00	0.914	1.689	0.211
480.00	0.555	2.051	0.256
1200.00	0.287	2.652	0.331

The right-hand column of Table 9.6 indicated the increase in concentration of DO in the lake due to the direct addition of oxygen in the saturated drops. For example, a 24-h rainfall at 4.8 mm/h increases the DO by about 0.11 mg/l.

The final topic to be considered is that relating to the total amount of oxygen transferred per unit time and area due to the two transfer mechanisms of rainfall. That is

$$M = M_1 + M_2 = C_1 r + K_r (C_s - C_2) \tag{9.3.7}$$

in which M is the total amount transferred, M_1 is the amount transferred by direct addition and M_2 is the amount transferred due to interfacial turbulence; C_1 and C_2 are, respectively, the concentrations of DO in the drops and in the well-mixed body of water. A quantity, p, is defined as follows

$$p = \frac{M_2}{M} = \frac{K_r (C_s - C)}{C_1 r + K_r (C_s - C)} \tag{9.3.8}$$

Letting $S_1 = C_1/C_s$ and $S_2 = C_2/C_s$, eq. *9.3.8* can be written in the following form

$$p = \frac{1}{1 + \left(\frac{r}{K_r}\right)\left(\frac{S_1}{1 - S_2}\right)} \tag{9.3.9}$$

Clearly, the quantity, p, represents the fraction of the oxygen transferred due to interfacial turbulence and the quantity, $1-p$, represents the fraction transferred by direct addition.

Results of some calculations are given in Table 9.7. The indicated values of p correspond to the values, $S_1 = 1.0$ (saturated water drops), and to three values of S_2 (0, 50 and 90 % saturated concentrations of DO in the lake). For example, during a 24 h, 4.8 mm/h rainfall, with the lake water at 50 % saturated concentration, about 66 % of the oxygen transfer is due to interfacial turbulence and approximately 34 % is due to direct addition. The results presented in table 9.7 are based on values of the oxygen transfer coefficient due to rainfall only. If the effect of wind is also involved, the total oxygen transfer coefficient, K, described by eq. *8.4.9*, must be employed. In this case, the magnitudes of K, and hence of p, are larger.

TABLE 9.7. FRACTIONS OF OXYGEN TRANSFER DUE TO INTERFACIAL TURBULENCE AND TO DIRECT ADDITION, CHAPALA LAKE

T h	r mm/h	K_r mm/h	r/K_r	p $S_1 = 1.0$ $S_2 = 0.0$	p $S_1 = 1.0$ $S_2 = 0.5$	p $S_1 = 1.0$ $S_2 = 0.0$
0.083	283	3 187	0.0888	0.918	0.849	0.530
0.167	171	1 689	0.1012	0.908	0.832	0.497
0.333	104	903	0.1152	0.897	0.813	0.465
0.50	77.8	626	0.1242	0.890	0.810	0.446
1.00	47.3	334	0.1416	0.876	0.779	0.414
2.00	28.7	178	0.1611	0.861	0.756	0.383
6.00	13.0	65.7	0.1979	0.835	0.716	0.336
12.00	7.9	36.1	0.2253	0.816	0.689	0.307
24.00	4.8	18.8	0.2559	0.796	0.661	0.281
48.00	2.91	9.97	0.2918	0.774	0.631	0.255
120.00	1.505	4.34	0.3468	0.743	0.590	0.224
240.00	0.914	2.32	0.3942	0.717	0.559	0.202
480.00	0.555	1.24	0.4495	0.690	0.527	0.182
1 200.00	0.287	0.54	0.5350	0.651	0.483	0.157

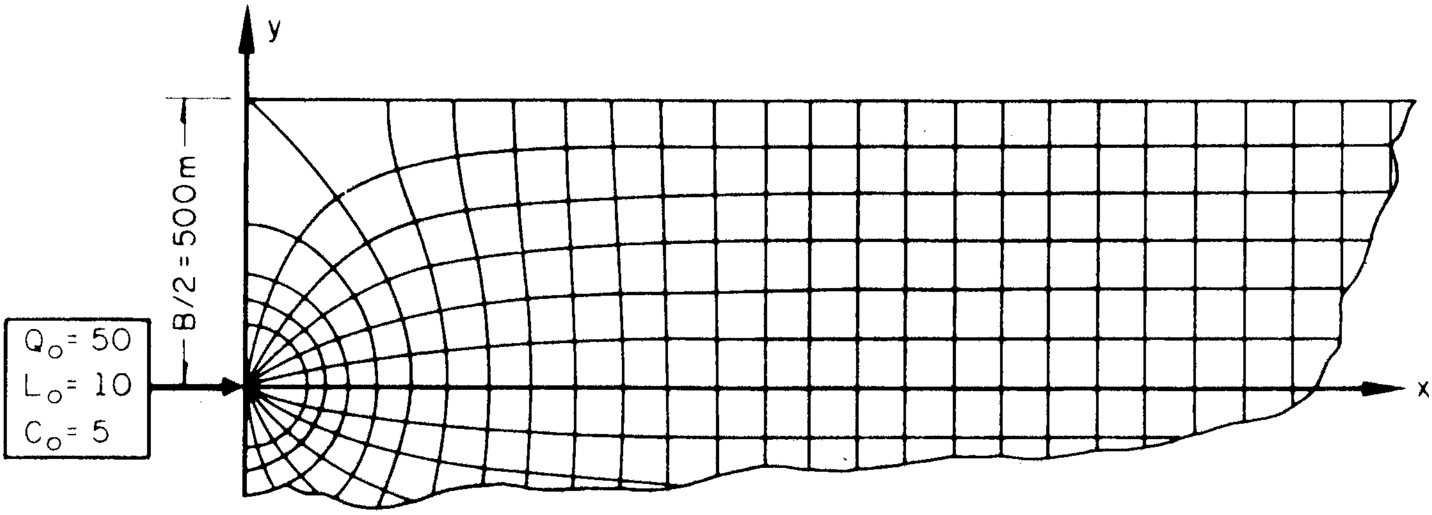

Fig 9.1. *Equipotentials and streamlines for the flow an infinite row of sources*

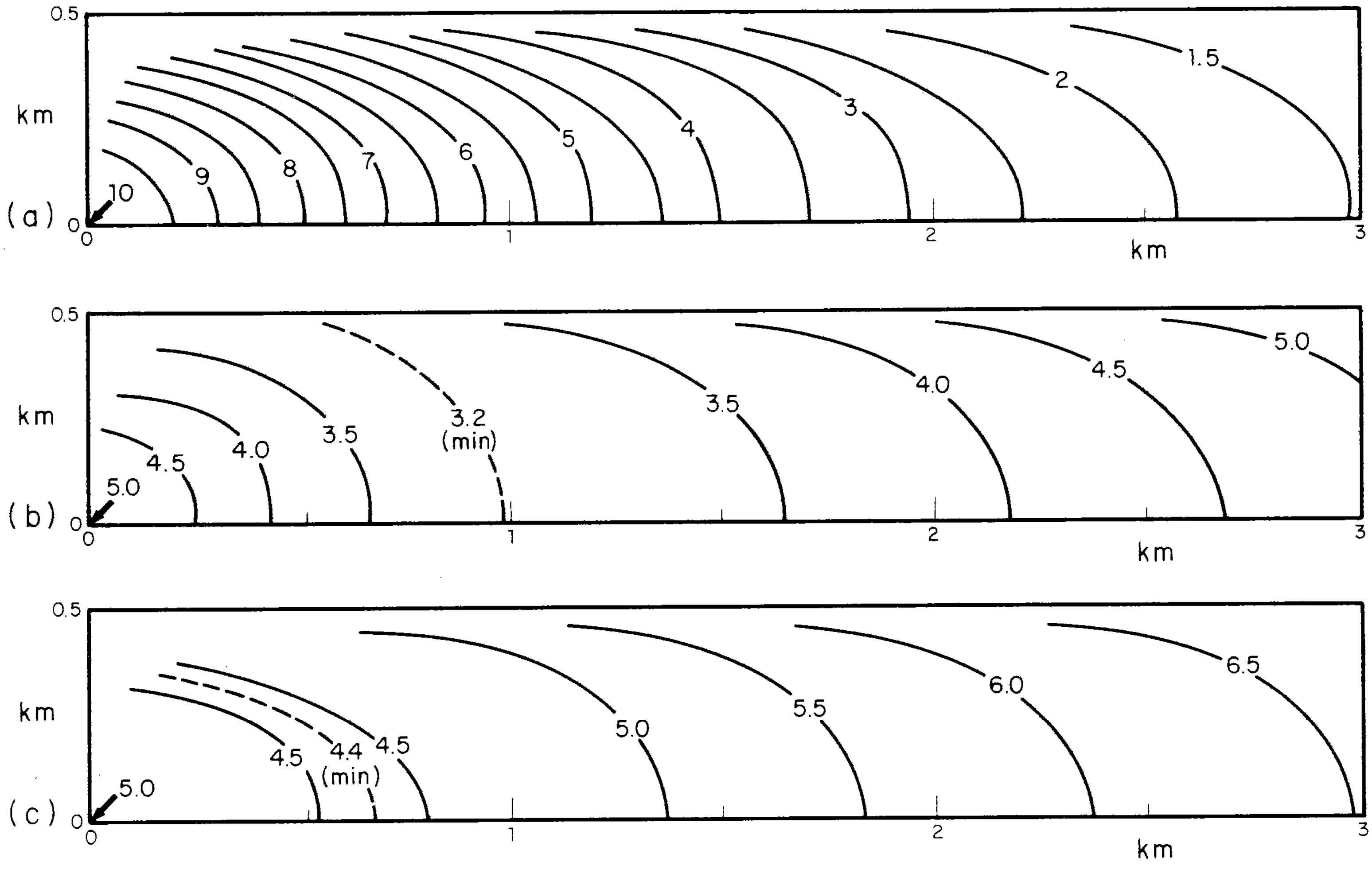

Fig 9.2. *Distributions of BOD and DO in the flow from an infinite row of sources (a) BOD for cases A and B, (b) DO for case A (wind only) and (c) DO for case B (wind and rain)*

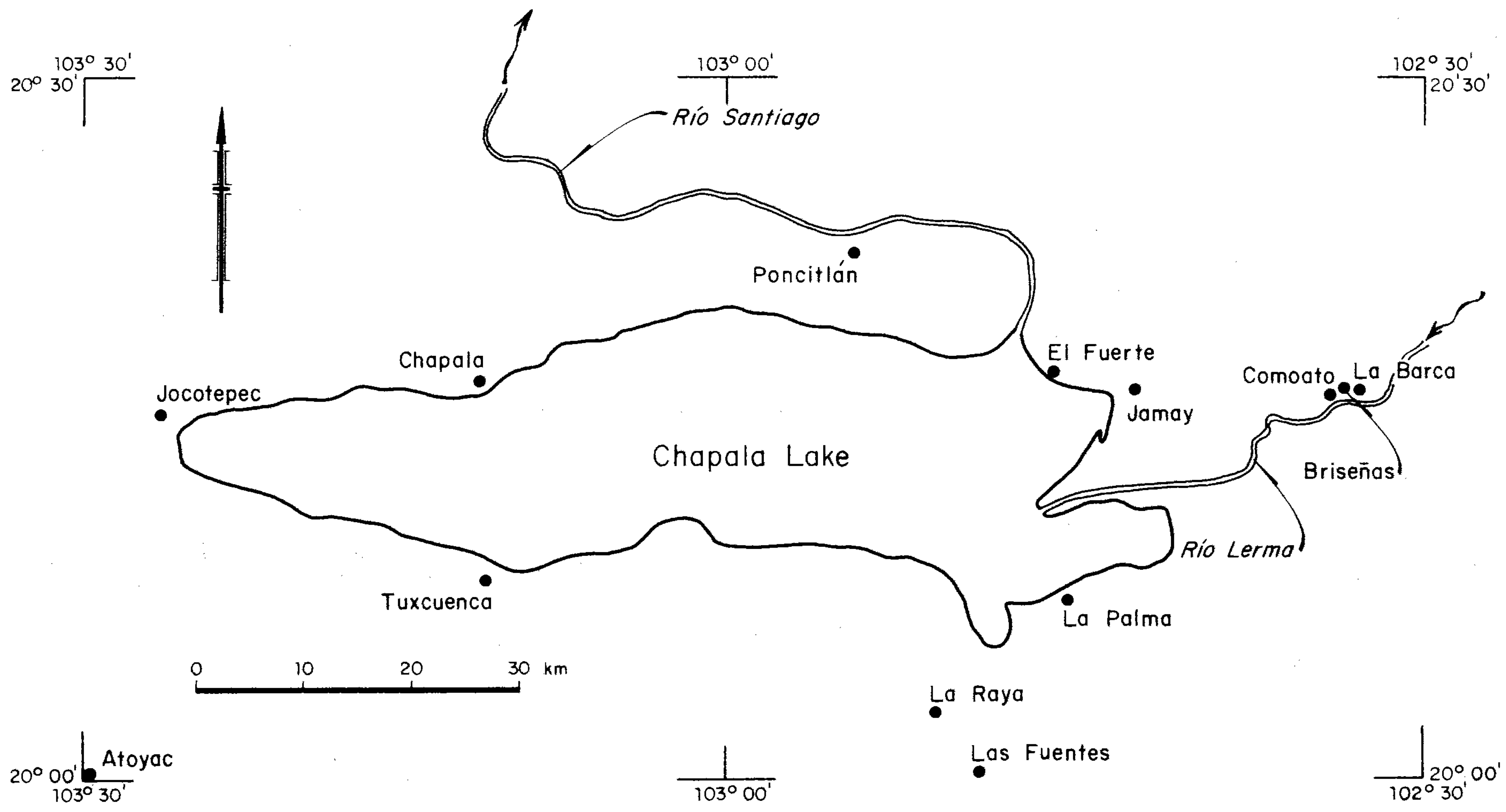

Fig 9.3. Climatological stations near Chapala Lake

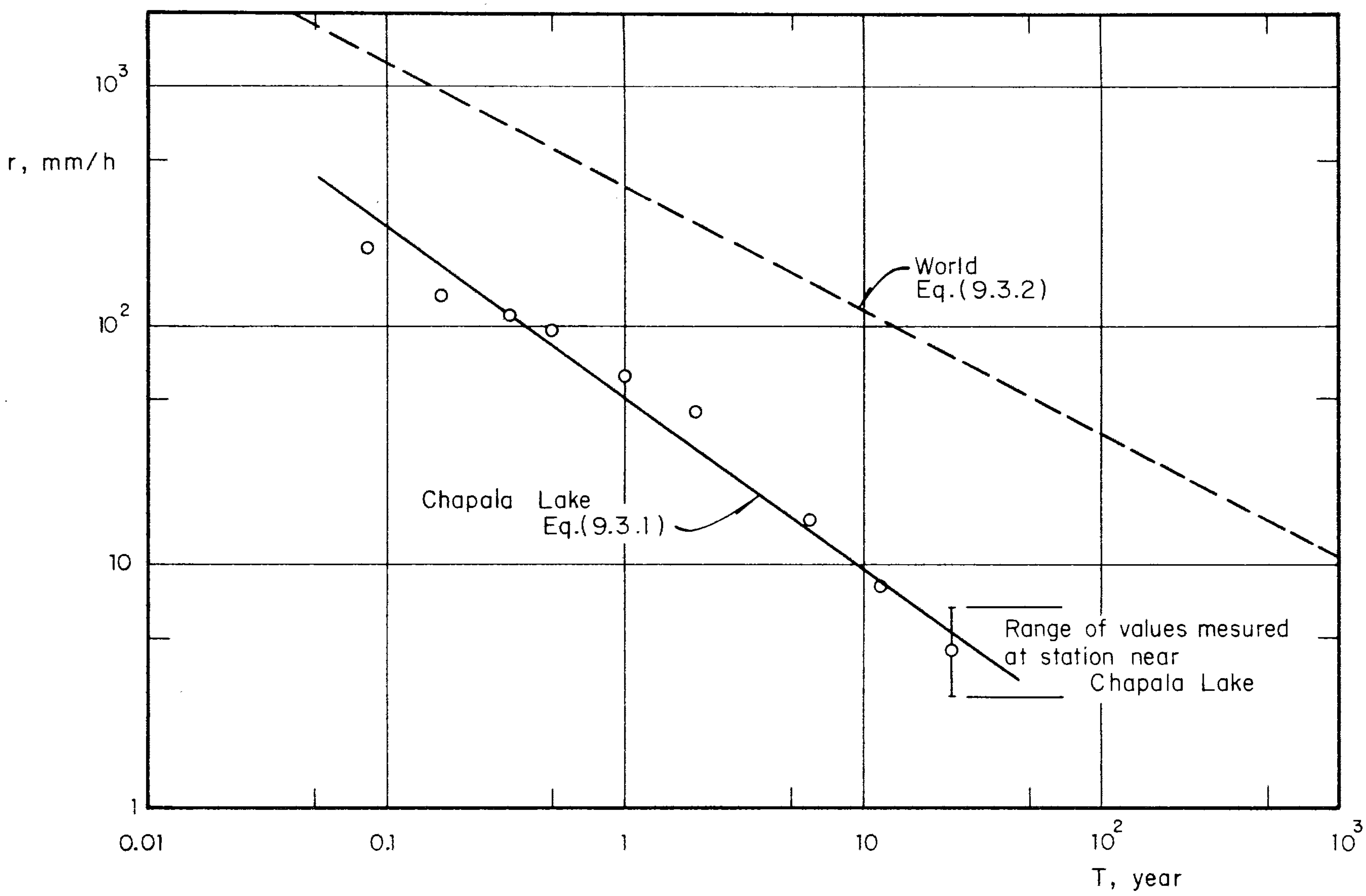

Fig 9.4. *Rainfall rates measured at Chapala Lake*

10. CONCLUSIONS AND RECOMMENDATIONS

10.1 Conclusions

The magnitude of the oxygen transfer coefficient in a lake or lagoon depends very much on the velocity of the wind. When the wind velocity is less than about 7 m/s, the transfer coefficient is approximately proportional to the velocity. When the velocity is greater than about 10 m/s, the transfer coefficient is approximately proportional to the square of the velocity.

The Reynolds numbers of water drops in typical rainfalls are larger than those corresponding to the range of validity of Stokes law. The fall velocity of a raindrop depends primarily on its diameter. The density of the air also affects the magnitude of the fall velocity. The reduced air density at high elevations causes an increase in the magnitude of the fall velocity of a drop and hence an increase in the kinetic energy of the drop.

The diameters of drops in a rainfall appear to be exponentially-distributed with a relatively large number of small diameter drops and a relatively small number of large diameter drops. As the rainfall rate is increased. The percentage of large diameter drops is increased.

For such an exponential distribution of drop diameters, the largest number of drops per unit volume corresponds to the smallest diameter range. Other rainfall parameters are the number of drops crossing unit horizontal area in unit time, the volumetric concentration, the rainfall rate, and the energy flux or power. Maximum values of these four parameters occur at successively increasing values of drop diameters.

The power of a rainfall whose drop diameters are exponentially-distributed is expressed by the equation

$$P = (0.239 + 0.103 \times 10^{-4} z)\, r^{1.26}$$

where P is expressed in $\mu J/s\text{-}cm^2$. The elevation, z, is given in meters and the rainfall rate, r, is expressed in mm/h.

The power of a rainfall whose drop diameters are uniform is given by the equation

$$P = \frac{10^{-1}}{2} \rho' r V^2$$

where ρ' is the density of water and V is the fall velocity. From this equation it is observed that the power is directly proportional to the rainfall rate for uniform drop-diameter rainfalls. Limited experimental data indicate that the oxygen transfer coefficient is also directly proportional to the rainfall rate for rainfalls with uniform size drops. Accordingly, it appears reasonable to make the tentative assumption that the transfer coefficient is directly proportional to the power of a rainfall.

The magnitude of the oxygen transfer coefficient due to the combined effects of wind and rain is less than the sum of the transfer coefficients due to the two effects considered separately.

Considerable information has been obtained and analyzed concerning rainfalls at Chapala Lake. A maximum-type rainfall of about 5 mm/h over a period of 24 hours produces a surface reaeration coefficient, $K_2 = 0.056\ d^{-1}$. A rainfall of approximately 8 mm/h over a period of 12 hours yields a value, $K_2 = 0.105\ d^{-1}$.

The concentration of dissolved oxygen in Chapala Lake is increased during the rainy months, June to September, not only by increased in the magnitude of the surface reaeration coefficient due to rainfall but also by direct addition of oxygen contained in the drops falling on the lake. Computations show that more-or-less steady rainfalls during a period of about two months can increase the concentration of DO in the lake by about 0.35 mg/l due to direct addition.

The results obtained in the present study may have an application in the very serious problem of erosion of soil due to rainfall.

10.2 Recommendations

The present research has been essentially a preliminary study of the effects of wind and rain on surface reaeration. It is recommended that this study be continued along the following lines.

Experimental apparatus should be designed and constructed to measure, separately and in combination, the effects of wind and rain on the oxygen transfer coefficient. It is especially important that additional information be obtained concerning the effect of rain. It is necessary to confirm or modify the assumption that the transfer coefficient is directly proportional to rainfall power.

Theoretical analyses might be carried out to examine various interaction phenomena between wind action and rainfall. For example, the wind surely affects the trajectories and energies of the drops. In turn, the drops probably influence the velocity distribution and roughness heights associated with the wind profile.

Further attention should be given to the subject of drop diameter distributions in artificial and natural rainfalls.

It would be desirable to conduct experiments in the field to determine values of surface reaeration coefficients due to natural winds and rainfalls.

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APPENDIX

TABLE A.1. EQUILIBRIUM CONCENTRATION OF DISSOLVED OXYGEN, C_s, mg/l

T, Celsius \ z, m	0	1 000	2 000	3 000	4 000	5 000
p, Pa	5.70	5.06	4.47	3.95	3.47	3.04
10	11.3	10.0	8.9	7.8	6.9	6.0
11	11.1	9.8	8.7	7.7	6.7	5.9
12	10.8	9.6	8.5	7.5	6.6	5.8
13	10.6	9.4	8.3	7.3	6.4	5.6
14	10.4	9.2	8.2	7.2	6.3	5.5
15	10.2	9.1	8.0	7.1	6.2	5.4
16	10.0	8.9	7.8	6.9	6.1	5.3
17	9.8	8.7	8.7	7.7	6.8	5.2
18	9.6	8.5	7.5	6.6	5.8	5.1
19	9.4	8.3	7.4	6.5	5.7	5.0
20	9.2	8.2	7.2	6.4	5.6	4.9
21	9.0	8.0	7.1	6.3	5.5	4.8
22	8.8	7.8	6.9	6.1	5.4	4.7
23	8.7	7.7	6.8	6.0	5.3	4.6
24	8.5	7.5	6.7	5.9	5.2	4.5
25	8.4	7.4	6.6	5.8	5.1	4.5
26	8.2	7.3	6.4	5.7	5.0	4.4
27	8.1	7.2	6.3	5.6	4.9	4.3
28	7.9	7.0	6.2	5.5	4.8	4.2
29	7.8	6.9	6.1	5.4	4.7	4.2
30	7.6	6.7	6.0	5.3	4.6	4.1

Note: The above values are for fresh water. The correction for salinity is applied by multiplying the above values by

$$(1 - S/100,000)$$

where S is the salinity in parts per million of chloride

Ref: Phelps, E B, **Stream sanitation**, John Wiley and Sons, New York (1944), 147

TABLE A.2. THE STANDARD ATMOSPHERE

Elevation, m	Pressure, N/cm²	Pressure, Pa	Density, g/cm³ x 10³	Temperature Celsius	Viscosity, dPa-s x 10⁵
0	10.13	5.70	1.226	15.0	1.78
500	9.55	5.37	1.168	11.7	1.77
1 000	8.99	5.06	1.112	8.5	1.75
1 500	8.45	4.76	1.059	5.2	1.73
2 000	7.95	4.47	1.007	2.0	1.72
3 000	7.01	3.95	0.910	−4.5	1.68
4 000	6.16	3.47	0.820	−11.0	1.62
5 000	5.40	3.04	0.736	−17.5	1.60
6 000	4.72	2.66	0.660	−24.0	1.58
7 000	4.14	2.33	0.593	−30.5	1.55
8 000	3.56	2.00	0.525	−37.0	1.51
9 000	3.10	1.75	0.469	−43.5	1.48
10 000	2.64	1.49	0.413	−50.0	1.45

Ref: Batchelor, G K, **An introduction to fluid dynamics**, Cambridge University Press, London (1967), 594-595